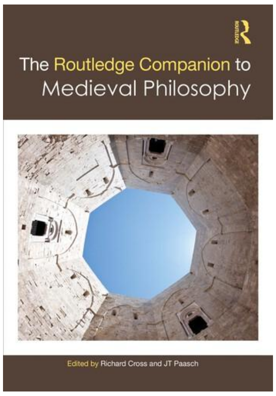

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LOGIC GAMES

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Thirteenth- and fourteenth-century scholastic writers describe a family of logic games they call *disputationes de obligationibus*, or just *obligationes* for short. We could anglicize the name to "disputations of obligation," but the idea is captured better with something like "games of logical commitment" or "games of logical obligation." I will call them "obligational games," or even just "games."

There are different variations, but all of them are two-player games, where one player puts forward a sequence of propositions to the other player, who must concede, deny, or doubt them, in accordance with the rules of the game. The proposing player tries to trick the other player into violating the rules, while the other player tries not to violate the rules.

The plan of this chapter is as follows. In the first section, I give a brief overview of how the games were played. In the second section, I describe one type of game in detail. In the third section, I discuss the origin and development of the games. In the fourth section, I survey various interpretations of the games. In the final section, I conclude with some pointers for further reading.

The Games

In the thirteenth and fourteenth centuries (and beyond), scholastic thinkers wrote treatises dedicated to obligational games. Many follow the same format. The author presents their version of the rules for the games, and then they discuss a series of examples involving sophisms. By using sophisms in their examples, the author aims to show that their version of the rules is good enough to handle even such hard cases as sophisms.

Despite the diversity of game variants present in the scholastic texts, from a high level, all of them proceed along similar lines. There are two players, called the opponent and the respondent, and play proceeds in a series of rounds.

- **Round zero**, i.e., the **thesis round** (establishing the thesis).
  - The opponent puts forward a proposition of their choosing. I will call this the thesis of the game.
  - The respondent must concede or deny the thesis (based on the particular rules of the game). If they concede it, the game begins. If they deny it, the game does not start.
- **Round 1**.
  - The opponent proposes another proposition of their choosing.
  - The respondent must concede it, deny it, or doubt it (based on the particular rules of the game).

- **Round 2, 3, and so on**.
  - The opponent can repeat these rounds as many times as they like, each time proposing a proposition of their choosing, which the respondent must concede, deny, or doubt (according to the rules).
- **Final round**.
  - When the opponent has had enough, they call out "*cedat tempus*," i.e., "time's up!" or "game over!"
  - The players (and perhaps any onlookers) review the respondent's responses to determine if they violated the rules at any point. If the respondent violated no rules, they win. If they violated the rules, the opponent wins.

Most of these games have certain structural components in common:

- *Players*. Each game has two players, namely an opponent and a respondent. I will abbreviate these as **O** and **R**, respectively.
- *A thesis*. Each game has a thesis, which I will abbreviate as **Θ**.
- *A thesis rule*. For each game, there is a rule which stipulates whether the respondent should concede or deny the proposed thesis.
- *A concede rule*. For each game, there is a rule which stipulates whether the respondent should concede, deny, or doubt any proposition proposed by the opponent after the thesis.
- *Conceded propositions*. During each game, the respondent may concede some collection of propositions proposed by the opponent. I will abbreviate this collection as **C** (short for "Conceded propositions").
- *Denied propositions*. During each game, the respondent may deny some collection of propositions proposed by the opponent. Rather than keep these in their own collection, I will just include them in **C** by negating them. For instance, if the respondent denies the proposition "the pope is sitting," then I will say that the respondent concedes its negation, i.e., they concede "it is not the case that the pope is sitting," which I will abbreviate as "¬(the pope is sitting)."
- *Doubted propositions*. During each game, the respondent may doubt some collection of propositions proposed by the opponent. I will abbreviate this collection as **D** (short for "Doubted propositions").
- *The context*. At various points during a game, the respondent may need to consult what they believe to be true or false about the real world. I will call this collection of beliefs the context of the game, and I will abbreviate it as **Γ**.

Different types of rules result in different types of games. In the thirteenth and early fourteenth centuries, six main types of games are discussed.

- *Positio*. In a *positio* game, the respondent must concede the thesis if he wants the game to start from it, and then concede any proposition the opponent proposes which logically follows from it. The opponent's goal is to try to get the respondent to concede inconsistent propositions.
- *Depositio*. In a *depositio* game, the respondent must deny the thesis if he wants the game to start from it. The respondent's task is then to deny any further propositions the opponent might propose which imply that the thesis is true.
- *Dubitatio*. In a *dubitatio* game, the respondent must treat the thesis as if it is not known whether it is true or false. The respondent's task is then to deny any propositions the opponent might propose which imply that the thesis is true or false.

- *Impositio*. In an *impositio* game, the opponent proposes that a word or phrase signifies something different than it normally does. For instance, the opponent might propose that the players agree that, for the duration of the game, the name "Socrates" signifies a donkey rather than the man.
- *Petitio*. In a *petitio* game, the opponent proposes that the respondent should always respond in a certain way. For instance, the opponent might propose that the respondent should always concede that Socrates is a donkey, whenever it is proposed.
- *Sit verum*. In a *sit verum* game, the opponent proposes that the players agree that a particular proposition be true for the duration of the game. For instance, the opponent might propose that "The pope is sitting" be taken as true during the course of the game.

During the scholastic period, this list is by no means accepted as standard. In the fourteenth century, some authors reduce the above list of game types to a smaller list, arguing that some of these game types are superfluous or could be reduced to others.

The *Positio* Game

The type of game that is discussed most by both medieval writers and modern scholars is the *positio* game. In this section, I discuss in detail two important versions of the *positio* game that we find in the fourteenth century.

The first is Walter Burley's version, from the very beginning of the fourteenth century. The second is Roger Swyneshead's version, from a few decades later. There are other versions besides these two, but a variety of medieval authors consider Burley's and Swyneshead's to be the two dominant versions of the *positio* game. Writing a little later in the fourteenth century, Robert Fland even calls Burley's version the *antiqua* variant and Swyneshead's the *nova* variant (see Spade 1980).

Burley's Positio Game

Burley published his treatise on *Obligationes* in 1302. For the Latin text, see Green (1963), and for a partial English translation, see Burley (1988).

Modern scholars often consider Burley's version of *positio* to be the standard variant of *positio* games (e.g., Spade 1982a: 4), and medieval thinkers seem to see it this way too. Scholastic writers after Burley often discuss his view, and some (like Roger Swyneshead) even seem to formulate their own versions in response.

Burley's version of *positio* can be summarized in the following way (for more detailed descriptions, see Spade 1982a; Stump 1989c; Yrjönsuuri 2001b; Dutilh Novaes 2005).

- *The context*. Burley considers the context **Γ** to be what the respondent knows about the actual world, although Burley allows the players to explicitly stipulate that certain things are true if they want to make the game more interesting. For convenience, I will pretend that before the game begins, the players always explicitly agree on **Γ**, i.e., they explicitly enumerate the propositions that the respondent will take to be true during the course of the game (even though in practice this may not always happen).
- *The thesis rule*. The respondent should concede any thesis **Θ** which is not self-contradictory. In practice, players tend to play with theses which are contingently false, since that makes for a more interesting game.
- *The concede rule*. For any proposition **P** that the opponent proposes, the respondent must first evaluate **P** against the collection of propositions **C** that they have already conceded. If **P**

logically follows from anything in **C**, then the respondent must concede **P**. If it contradicts anything in **C**, then the respondent must deny it. If **P** neither follows from nor contradicts anything in **C**, then it is inferentially irrelevant. In that case, the respondent must turn to the context **Γ**. If **P** is contained in **Γ**, then the respondent must concede it, if it contradicts anything in **Γ**, they must deny it, and if **Γ** says nothing about **P**, they must doubt it.

- *Winning.* When the game ends, the respondent wins if **C** is consistent. The opponent wins if **C** contains any two propositions that contradict each other.
- *Evaluation.* Answers should be evaluated as if the game takes place at a single (not necessarily determinate) moment of time. Hence, changes in the world over time are not considered. For instance, the proposition "you are sitting" cannot be true in one round of the game because the respondent is then sitting, and then be false in a later round of the game because the respondent is then standing. Instead, the players should evaluate all propositions from a single moment in time (whether the respondent is sitting or not should thus be explicitly fixed by stating it in **Γ**).

I should emphasize that these are rules for Burley's version of the *positio* game only. Other authors provide alternative rules, as we shall see, for example, with Roger Swyneshead. Also, these rules would look different if we were discussing a different type of game. For instance, if the game at hand were *dubitatio*, the *concede rule* would look quite different.

An Example

Consider a simple example. Suppose the players are students at Oxford. The following table encodes a game that they might play, with explanatory commentary appended below each round.

Round	*Opponent*	*Respondent*
[Γ]	**Γ = { R is in Oxford, R is a student }** The players first establish the context **Γ**—i.e., a list of the propositions they agree are believed by the respondent **R** to be true. In this case, it is that **R** is in Oxford, and **R** is a student. At this point in the game, **R** has conceded no propositions, and doubted no propositions, so **C** and **D** are empty: • **C**: { } • **D**: { }	**n/a**
[Θ]	**I posit that: you are in Rome and you are the pope.** Here the opponent **O** proposes a thesis **Θ**, namely a conjunction of two things: that the respondent **R** is in Rome, and that **R** is the pope. This is contingently false, because **R** in fact is in Oxford (rather than Rome), and is not the pope. However, it is not self-contradictory, so **R** concedes it. **R** has now conceded one proposition, namely the thesis **Θ**: • **C**: { **R** is in Rome and **R** is the pope } • **D**: { }	**I concede it.**
[1]	**I propose that you are the pope.** In this round, **O** proposes another proposition, namely that **R** is the pope. This follows from what **R** has already conceded in **C**. After all, if one has conceded a conjunction (i.e., that **R** is in Rome *and* that **R** is the pope), then one must concede each conjunct individually too (e.g., that **R** is the pope). So **R** concedes this too.	**I concede it.**

	Now **R** has conceded two propositions, namely the thesis **Θ**, and the proposition proposed here in round [1]: • **C**: { **R** is in Rome and **R** is the pope, **R** is the pope } • **D**: { }	
[2]	**Time is up.** At this point, **O** has had enough and finishes the game by calling out "time is up." The winner of this game is **R**, because **R** has kept **C** consistent.	**n/a**

This illustrates basic gameplay. First, there is an initial (at least hypothetical) context round, where the players establish the context of the game. Next, there is a thesis round, where the players negotiate a thesis. If a thesis is accepted, the game begins. Following that, there are subsequently numbered rounds (round 1, round 2, and so on), until the opponent ends the game by calling out "time is up."

The Respondent Denies the Thesis

As noted earlier, the respondent should deny any thesis that is self-contradictory, in which case the game does not start. Here is an example.

Round	*Opponent*	*Respondent*
[Γ]	**Γ = { R is in Oxford, R is a student }** The players first establish the context **Γ**.	**n/a**
[Θ]	**I posit that you are in Rome and you are not in Rome at the same time.** The opponent **O** proposes a thesis **Θ**, namely that the respondent **R** is both in Rome and not in Rome at the same time. Since this is self-contradictory, **R** denies it. Thus, the game does not begin.	**I deny it.**

Notice that since the respondent can deny the proposed thesis, there is some burden on the opponent to be strategic and propose a thesis that has a good chance of being accepted.

The Respondent Loses

If the respondent fails to see a contradiction, they lose. Here is an example.

Round	*Opponent*	*Respondent*
[Γ]	**Γ = { R is in Oxford, R is a student }** The players first establish the context **Γ**.	**n/a**
[Θ]	**I posit that you are in Rome.** The opponent **O** proposes a thesis **Θ**, which **R** concedes. • **C**: { **R** is in Rome } • **D**: { }	**I concede it.**

[1]	**I propose that you are not in Rome.** Now **O** proposes another proposition, which is the opposite of the thesis. **R** concedes it (perhaps **R** is daydreaming and not thinking carefully about what they are doing). • **C**: { **R** is in Rome, ¬(**R** is in Rome) } • **D**: { }	**I concede it.**
[2]	**Time is up.** **O** ends the game. The players (and perhaps any onlookers) examine **C**, and they see that **R** has conceded contradictory propositions. Hence, **R** loses, and **O** wins.	**n/a**

An Irrelevant Proposition

As noted earlier, if a proposed proposition is inferentially irrelevant to **C**, the respondent must evaluate it against the context. Here is an example.

Round	*Opponent*	*Respondent*
[Γ]	**Γ = { R is in Oxford, R is a student }** The players first establish the context **Γ**.	**n/a**
[Θ]	**I posit that you are in Rome.** The opponent **O** proposes a thesis **Θ**, which **R** concedes. • **C**: { **R** is in Rome } • **D**: { }	**I concede it.**
[1]	**I propose that you are a student.** Now **O** proposes that **R** is a student. This is inferentially irrelevant to **C**, because it neither follows from nor contradicts anything contained in **C**. After all, one can be in Rome, whether they are a student or not. Since it is irrelevant, **R** has to turn to the context **Γ** to determine if they should concede, deny, or doubt it. The proposition is indeed contained in **Γ**, so **R** concedes it: • **C**: { **R** is in Rome, **R** is a student } • **D**: { }	**I concede it.**
[2]	**Time is up.** **O** ends the game, and **R** wins, because **C** is consistent.	**n/a**

Doubting a Proposition

If the context says nothing about a proposed proposition, then the respondent must doubt that proposition. Here is an example.

Round	*Opponent*	*Respondent*
[Γ]	**Γ = { R is in Oxford, R is a student }** The players first establish the context **Γ**.	**n/a**
[Θ]	**I posit that you are in Rome.**	**I concede it.**

The opponent **O** proposes a thesis **Θ**, which **R** concedes.

- **C**: { **R** is in Rome }
- **D**: { }

[1] **I propose that the pope is sitting.** — **I doubt it.**

Now, **O** proposes that the pope is sitting. This is inferentially irrelevant to **C**, so **R** has to evaluate it against the context **Γ**.
Since the context says nothing about whether the pope is sitting or not, **R** must mark this proposition as one that he does not know and hence doubts:

- **C**: { **R** is in Rome }
- **D**: { The pope is sitting }

[2] **Time is up.** — **n/a**

O ends the game, and **R** wins, because **C** is consistent.

Changing Responses

There is nothing to prevent the opponent from putting forward the same proposition multiple times in a game. Moreover, under Burley's rules, the opponent can force the respondent to first doubt a proposition, and then concede it. Here is an example.

Round	*Opponent*	*Respondent*
[Γ]	**Γ = { ¬(It is raining) }** The players first establish the context **Γ**. In this case, they agree that it is not raining.	**n/a**
[Θ]	**I posit that the pope is sitting or it is raining.** **O** proposes a thesis **Θ**, which is a disjunction of (i) something unknown (the pope is sitting), and (ii) the opposite of something believed to be true i.e., it is raining, which is the opposite of ¬(it is raining). Since that is not self-contradictory, **R** concedes it. • **C**: { The pope is sitting or it is raining } • **D**: { }	**I concede it.**
[1]	**I propose that the pope is sitting.** Next, **O** proposes that the pope is sitting. This is inferentially irrelevant to **C**. The only thing contained in **C** is the thesis **Θ**, which implies neither that the pope is nor is not sitting (**Θ** says only that the pope is sitting or it is raining, but it does not say which). So **R** has to evaluate this against the context **Γ**. **Γ** says nothing about whether the pope is sitting, so **R** must doubt this proposition. • **C**: { The pope is sitting or it is raining } • **D**: { The pope is sitting }	**I doubt it.**
[2]	**I propose that it is raining.** Now, **O** proposes that it is raining. This is also inferentially irrelevant to **C**, for the same reason that [1] was, so **R** has to evaluate this against the context **Γ** too. This is false in **Γ**, so **R** must deny that it is raining. • **C**: { The pope is sitting or it is not raining, ¬(It is raining) } • **D**: { The pope is sitting }	**I deny it.**

[3]	**I propose that the pope is sitting.** For the second time, **O** proposes that the pope is sitting. This time, it follows from **C**. There are two propositions contained in **C**: the thesis **Θ** (which says either the pope is sitting or it is raining), and the negation of [2] (i.e., it is not raining). By disjunctive syllogism, it follows that the pope must be sitting. So, **R** must concede this proposition, which he doubted before. • **C**: { The pope is sitting or it is not raining, ¬(It is raining), The pope is sitting } • **D**: { }	**I concede it.**
[4]	**Time is up.** **O** ends the game, and **R** wins, because **C** is consistent.	**n/a**

Notice that in this game, the respondent first doubts a proposition and then concedes it. In particular, in round [1], they doubt that the pope is sitting, and then in [3], they concede it.

This game pattern can be generalized. The template for any game that follows this pattern has the following shape (where **P** and **Q** are any distinct arbitrary propositions):

Round	*Opponent*	*Respondent*
[Γ]	**Γ = { ¬(Q) }**	**n/a**
[Θ]	**I posit that P or Q.**	**I concede it.**
[1]	**I propose that P.**	**I doubt it.**
[2]	**I propose that Q.**	**I deny it.**
[3]	**I propose that P.**	**I concede it.**
[4]	**Time is up.**	**n/a**

Nevertheless, a change from doubting to conceding is the only way that the respondent can change their response. For under Burley's rules, once a respondent concedes or denies a proposition, they cannot give a different response if it is proposed again (see Spade 1982a; Dutilh Novaes 2005).

Notice that matters are different for other types of games. For example, in a *dubitatio* game, once propositions are placed in **D**, they can never move out of **D**, since *dubitatio* revolves around propositions that are doubted.

Order Matters

The order in which the opponent proposes propositions matters. In the previous game, the respondent was able to first doubt that the pope is sitting, but in the following game, they are not given this chance, because the opponent proposes the propositions in a different order.

Round	*Opponent*	*Respondent*
[Γ]	**Γ = { ¬(It is raining) }** The players first establish the context **Γ**.	**n/a**
[Θ]	**I posit that the pope is sitting or it is raining.** **O** proposes the thesis **Θ**, which **R** concedes. • **C**: { The pope is sitting or it is raining } • **D**: { }	**I concede it.**

[1]	**I propose that it is raining.** Now **O** proposes that it is raining. This is inferentially irrelevant to **C**, for the same reason it was before, so **R** has to turn to **Γ**. Since it is false in **Γ**, **R** must deny it. • **C**: { The pope is sitting or it is raining, ¬(It is raining) } • **D**: { }	**I deny it.**
[2]	**I propose that the pope is sitting.** Now, **O** proposes that the pope is sitting. It follows from **C**, by disjunctive syllogism, so **R** must concede it. • **C**: { The pope is sitting or it is raining, ¬(It is raining), The pope is sitting } • **D**: { }	**I concede it.**
[3]	**Time is up.** **O** ends the game, and **R** wins, because **C** is consistent.	**n/a**

Granting any Proposition

Under Burley's rules, the opponent can force the respondent to grant almost any arbitrary proposition. Here is an example where the opponent forces the respondent to grant that (say) the pope is sitting.

Round	*Opponent*	*Respondent*
[Γ]	**Γ = { ¬(R is in Rome), R is a student }** The players first establish the context **Γ**.	**n/a**
[Θ]	**I posit that you are in Rome.** **O** proposes a thesis **Θ**, which **R** concedes. • **C**: { **R** is in Rome } • **D**: { }	**I concede it.**
[1]	**I propose that you are not in Rome, or the pope is sitting.** Here **O** proposes a disjunction of (i) something believed to be true but which contradicts the thesis (namely, that **R** is not in Rome), and (ii) something unknown (namely, that the pope is sitting). To evaluate this, **R** turns to the first disjunct, namely that **R** is not in Rome. This contradicts the thesis **Θ**, but that does not mean the whole proposition contradicts the thesis. After all, the second disjunct might be compatible with the thesis. So **R** turns to the second disjunct, namely that the pope is sitting. This is inferentially irrelevant to **Θ**, however, and so the whole proposition is inferentially irrelevant. Hence, **R** must turn to the context **Γ**. To evaluate the proposition against **Γ**, **R** again turns to the first disjunct, namely that **R** is not in Rome. This is in **Γ**, and a disjunction must be conceded whenever one of its disjuncts is true, so **R** concedes the whole disjunction. • **C**: { **R** is in Rome, ¬(**R** is in Rome) or the pope is sitting } • **D**: { }	**I concede it.**
[2]	**I propose that the pope is sitting.**	**I concede it.**

	Now **O** proposes that the pope is sitting. This follows from **C**. For **R** has conceded that (i) either **R** is not in Rome or the pope is sitting, and (ii) that **R** is in Rome. By disjunctive syllogism, it follows that the pope is sitting. So **R** must concede it. • **C**: { **R** is in Rome, ¬(**R** is in Rome) or the pope is sitting, The pope is sitting } • **D**: { }	
[3]	**Time is up.** **O** ends the game, and **R** wins, because **C** is consistent.	**n/a**

This game pattern can also be generalized. A game following this pattern will have the following shape (where **P** and **Q** are any distinct arbitrary propositions).

Round	*Opponent*	*Respondent*
[Γ]	**Γ = { ¬(P) }**	**n/a**
[Θ]	**I posit that P.**	**I concede it.**
[1]	**I propose that ¬(P) or Q.**	**I concede it.**
[2]	**I propose that Q.**	**I concede it.**
[3]	**Time is up.**	**n/a**

Contradictories in Different Games

Under Burley's rules, an opponent can force the respondent to grant some proposition in one game, and then in another game, they can force the respondent to grant its contradictory. Here is an example (modeled after Ashworth 1993: 370). First, the opponent forces the respondent to grant (say) that the respondent is sitting:

Round	*Opponent*	*Respondent*
[Γ]	**Γ = { R is human, R is standing }** The players first establish the context **Γ**.	**n/a**
[Θ]	**I posit that every human is sitting.** **O** proposes a thesis **Θ**, which **R** concedes. • **C**: { Every human is sitting } • **D**: { }	**I concede it.**
[1]	**I propose that you are a human.** Here **O** proposes that **R** is a human. Since this is irrelevant, **R** turns to **Γ**, where it holds, so **R** concedes it. • **C**: { Every human is sitting, **R** is human } • **D**: { }	**I concede it.**
[2]	**I propose that you are sitting.** Here **O** proposes that **R** is sitting. This follows from **C**, so **R** concedes it. • **C**: { Every human is sitting, **R** is human, **R** is sitting } • **D**: { }	**I concede it.**
[3]	**Time is up.** **O** ends the game, and **R** wins, because **C** is consistent.	**n/a**

Then, in another game, the opponent starts with the same thesis but forces the respondent to grant the contradictory of what they granted before, namely that they are not sitting:

Round	*Opponent*	*Respondent*
[Γ]	**Γ = { R is human, R is standing }** The players first establish the context **Γ**.	**n/a**
[Θ]	**I posit that every human is sitting.** **O** proposes the same thesis **Θ**, which **R** concedes. • **C**: { Every human is sitting } • **D**: { }	**I concede it.**
[1]	**I propose that you are sitting.** Here **O** proposes that **R** is sitting. This is irrelevant (it has not been conceded that **R** is a human, so the proposition "**R** is a human" is not in **C**). Hence, **R** turns to **Γ**, where it is false that **R** is sitting, so **R** denies it. • **C**: { Every human is sitting, ¬(**R** is sitting) } • **D**: { }	**I deny it.**
[2]	**Time is up.** **O** ends the game, and **R** wins, because **C** is consistent.	**n/a**

Swyneshead's Positio Game

Roger Swyneshead wrote during the fourteenth century (d. 1365), and is not to be confused with the Oxford Calculator Richard Swyneshead. Roger published his treatise on *Obligationes* somewhere between 1330 and 1335. For the Latin text, see Spade (1977), and for the dates, see Weisheipl (1964).

In this treatise, Swyneshead presents his own version of the *positio* game, which differs from Burley's in important ways. According to some, the fact that Swyneshead's rules so exactly contradict Burley's on certain points suggests that Swyneshead probably formulated his rules in reaction to Burley. See, for example, Spade (1982a, 1982b), Stump (1982, 1989d), and Sinkler (1992).

Swyneshead's version of the *positio* game can be summarized in the following way (for more detailed descriptions, see Spade 1982a; Stump 1989d; Yrjönsuuri 2001b; and Dutilh Novaes 2006a).

- *The context.* Under Burley's rules, the context **Γ** is fixed and does not vary from round to round. By contrast, Swyneshead allows the context to change over time to reflect the actual state of affairs at each round of the game. Hence, for Swyneshead, the context **Γ** is not a static set of propositions that the players establish before the game begins, but rather a set of beliefs that the players establish (at least hypothetically, though probably not in practice) at each round. Hence, for Swyneshead, **Γ** is reset/re-established at each round.
- *The thesis rule.* Swyneshead says the respondent should concede any thesis that is contingent, or more exactly, any proposition that can change from being true to false (or vice versa) over time, when evaluated outside of the game.
- *The concede rule.* Under Burley's rules, the respondent must compare each proposed proposition against all propositions they have already conceded. By contrast, Swyneshead says the respondent must compare each proposed proposition only against the thesis **Θ**. Hence, for Swyneshead, **C** is reset at each round, so to speak, and can only be filled with a pair: the thesis, and the proposed proposition (if the respondent concedes it).

- *Winning*. When the game is over, the respondent wins if they have conceded no proposition that is inconsistent with the thesis. If they fail at this, the opponent wins.

Swyneshead's rules give his version of *positio* some interesting properties. I illustrate some of them in what follows.

The Context Changes

As noted a moment ago, under Swyneshead's rules, the context **Γ** can change from round to round. This means that the respondent can first concede and then deny a proposition, due to changes in the context. Here is an example.

Round	*Opponent*	*Respondent*
[Θ]	**Γ = { R is sitting }** **I posit that you are in Rome.** **O** proposes a thesis **Θ**, which **R** concedes. During this round, **R** is sitting, which is reflected in the context **Γ**. • **C**: { **R** is in Rome }	**I concede it.**
[1]	**Γ = { R is sitting }** **I propose that you are sitting.** **O** proposes that **R** is sitting, which is irrelevant to **Θ**, so **R** turns to the current context **Γ**. There **R** is sitting, so **R** concedes it. • **C**: { **R** is in Rome, **R** is sitting }	**I concede it.**
[2]	**Γ = { R is standing }** **I propose that you are sitting.** In the last round, **R** was sitting, but in this round **R** is now standing (which is reflected in the current context **Γ**). Here **O** proposes that **R** is sitting, and since that is irrelevant to **Θ**, **R** must turn to **Γ**. But during this round, it is false in **Γ** that **R** is sitting, so **R** denies it. • **C**: { **R** is in Rome, ¬(**R** is sitting) }	**I deny it.**
[3]	**Time is up.** **O** ends the game. **R** wins, because **R** conceded no proposition that contradicts the thesis **Θ**.	**n/a**

Notice that at each round, the context **Γ** is updated to reflect the current state of the world. In round [1], the respondent is sitting, which is reflected in the context during that round. Then at round [2], things change. The respondent stands up, and the context for that round is updated to reflect that.

Notice also that the set of conceded propositions **C** gets reset at each round. At each round, all previously conceded propositions are removed from **C**, except for the thesis. So at each round, the respondent is responsible for considering whether the proposed proposition follows only from the thesis (not from any other previously conceded propositions).

Notice finally that during the game, the respondent granted both that they are, and are not, sitting. Under Burley's rules, this would mean the respondent failed, since respondents are evaluated on the whole set of propositions that they concede in the game. Not so under Swyneshead's rules. Under Swynehead's rules, **C** is reset at each round, and so the respondent's answers are evaluated separately at each round. In the round [1], **C** is consistent, and then separately, in round [2], **C** is again consistent.

The Order Does Not Matter

As noted earlier, under Burley's rules, the order in which the opponent presents their propositions to the respondent matters. This is not so for Swyneshead's rules. At each round, the respondent concedes an inferentially relevant proposition only if it follows from the thesis alone, so they will always answer the same, no matter what order it comes in. Inferentially irrelevant propositions are similar: the respondent concedes them if they hold in the context, so for such propositions, the respondent will also always answer the same (in the same context), no matter what order it comes in. Consider the following game, played by Swyneshead's rules.

Round	*Opponent*	*Respondent*
[Θ]	**Γ = { ¬(It is raining) }** **I posit that the pope is sitting or it is raining.** **O** proposes a thesis **Θ**, which **R** concedes.	**I concede it.**
[1]	**Γ = { ¬(It is raining) }** **I propose that the pope is sitting.** **O** proposes that the pope is sitting, which is irrelevant to **Θ**, so **R** turns to **Γ**, which says nothing about it. Hence, **R** doubts it.	**I doubt it.**
[2]	**Γ = { ¬(It is raining) }** **I propose that it is raining.** **O** proposes that it is raining, which is irrelevant to **Θ**, so **R** turns to **Γ**. There it is false, so **R** denies it.	**I deny it.**
[3]	**Time is up.** **O** ends the game. **R** wins, because **R** granted nothing that contradicts **Θ**.	**n/a**

The respondent must doubt the proposition "the pope is sitting" when it is proposed in round [1]. This would be so under Burley's rules too.

Now consider the same game (again played by Swyneshead's rules), but with rounds [1] and [2] presented in reverse order.

Round	*Opponent*	*Respondent*
[Θ]	**Γ = { ¬(It is raining) }** **I posit that the pope is sitting or it is raining.** **O** proposes a thesis **Θ**, which **R** concedes.	**I concede it.**
[1]	**Γ = { ¬(It is raining) }** **I propose that it is raining.** **O** proposes that it is raining, which is irrelevant to **Θ**, so **R** turns to **Γ**. There it is false, so **R** denies it.	**I deny it.**
[2]	**Γ = { ¬(It is raining) }** **I propose that the pope is sitting.** **O** proposes that the pope is sitting, which is irrelevant to **Θ**, so **R** turns to **Γ**, which says nothing about it. Hence, **R** doubts it.	**I doubt it.**
[3]	**Time is up.** **O** ends the game. **R** wins, because **R** granted nothing that contradicts **Θ**.	**n/a**

This game is played by Swyneshead's rules, and we can see that the respondent doubts the proposition "the pope is sitting" when it is proposed in round [2], just as they did before when it was presented in round [1].

However, if the players played with Burley's rules, the respondent would have to concede this proposition in round [2], since they would have to consider both the thesis **Θ** and the proposition they granted in round [1]. Under Swyneshead's rules, however, the respondent can ignore round [1], so that they respond to the proposition in the same way they did before, no matter which round it is proposed in.

Granting the Conjuncts but Not the Conjunction

Under Swyneshead's rules, the opponent can force the respondent to grant two conjuncts but not their conjunction. Here is an example.

Round	*Opponent*	*Respondent*
[Θ]	**Γ = { ¬(R is in Rome), R is sitting }** **I posit that you are in Rome.** **O** proposes a thesis **Θ**, which **R** concedes.	**I concede it.**
[1]	**Γ = { ¬(R is in Rome), R is sitting }** **I propose that you are sitting.** This is irrelevant to **Θ**, so **R** must turn to **Γ**. There it is true, so **R** concedes it.	**I concede it.**
[2]	**Γ = { ¬(R is in Rome), R is sitting }** **I propose that you are in Rome and you are sitting.** Now, **O** proposes the conjunction of **Θ** and [1]. This conjunction is irrelevant to **Θ**. A conjunction will follow from a thesis only if both conjuncts follow from it. But here, **Θ** does not imply the second conjunct, namely whether **R** is sitting. So, this conjunction does not follow from **Θ**, and hence it is irrelevant. Consequently, **R** must turn to **Γ**, where the first conjunct is false, and so **R** denies it.	**I deny it.**
[3]	**Time is up.** **O** ends the game. **R** wins, because **R** granted nothing that contradicts **Θ**.	**n/a**

Notice how the respondent concedes that they are in Rome (in the thesis round), and that they are sitting (in round [1]). But then, when the conjunction of the two is proposed in round [2], the respondent must deny it.

Granting a Disjunction but Not the Disjuncts

Swyneshead's rules let the opponent force the respondent to grant a disjunction, and then deny both of its disjuncts. Here is a simple example.

Round	*Opponent*	*Respondent*
[Θ]	**Γ = { ¬(R is in Rome), ¬(R is sitting) }** **I posit that you are in Rome or you are sitting.** **O** proposes a thesis **Θ**, which **R** concedes.	**I concede it.**

[1]	**Γ = { ¬(R is in Rome), ¬(R is sitting) }** **I propose that you are in Rome.** **O** proposes that **R** is in Rome. This is irrelevant to **Θ**, because **Θ** implies neither that **R** is, nor is not, in Rome (**Θ** says only that **R** is in Rome or is sitting, but it does not say which). Hence, **R** must turn to the context **Γ**, where it is false, so **R** denies it.	**I deny it.**
[2]	**Γ = { ¬(R is in Rome), ¬(R is sitting) }** **I propose that you are sitting.** Now, **O** proposes that **R** is sitting. This is irrelevant to **Θ**, for the same reason that [1] was. Hence, **R** must again turn to **Γ**, where it is false, so **R** must deny it.	**I deny it.**
[3]	**Time is up.** **O** ends the game. **R** wins, because **R** granted nothing that contradicts **Θ**.	**n/a**

Notice how the respondent first concedes a disjunction (proposed as the thesis), but then they deny each of the disjuncts in successive rounds.

The Development of the Games

Treatises on obligational games survive from the thirteenth and fourteenth centuries (and beyond). If you look in the modern scholarship, you may come across a fairly common description of the development of this genre which runs as follows. Burley's treatise represents the "standard" thirteenth- and early fourteenth-century view. Then due to some criticisms from Richard Kilvington and others, a subtle shift in thinking begins, which culminates in the view of Swyneshead. Later medieval writers follow either Burley's or Swyneshead's version, though of course, they alter the rules in a variety of small ways. See Stump (1982), Spade (1982a, 1982b), and Yrjönsuuri (2001b).

The allegedly earliest thirteenth-century treatises on obligational games are edited in De Rijk (1974). These are undated, but De Rijk speculates that they must come from the early part of the thirteenth century. Other extant treatises from the thirteenth century that are worth mentioning include those edited in De Rijk (1975, 1976), and a treatise attributed to Nicholas of Paris (edited in Braakhuis 1998). There is also a treatise attributed to William of Sherwood (edited in Green 1963 and discussed in Stump 1989b), although some believe this text to actually be an early draft of Burley's (see Spade and Stump 1983).

In the fourteenth century, there are of course the treatises of Burley and Swyneshead. But other fourteenth-century treatises that are worth mentioning include the chapters on obligational games in Ockham's *Summa Logicae* (Ockham 1974), a passage in Richard Kilvington's *Sophismata* (1990), and treatises by Richard Lavenham (Spade 1978), Robert Fland (Spade 1980), Richard Brinkley (1995), John of Holland (1985), and Paul of Venice (1988). Ralph Strode's treatise on obligational games is not edited, but it is discussed in Ashworth (1993) and Dutilh Novaes (2006b).

What is the source or original inspiration for the medieval obligational game? It is tempting to look to Aristotle. In the *Topics* books I and VIII, Aristotle describes a dialectical logic game with two players, called the proponent and the opponent. The proponent first poses a thesis, after which the opponent asks a series of questions, with the aim of getting the proponent to grant contradictory propositions that follow from their thesis.

The similarity of Aristotle's games to scholastic obligational games is striking. Unfortunately, it does not seem that scholastic writers paid much attention to *Topics* I and VIII until somewhere in the middle of the thirteenth century, at which point obligational games had already appeared.

So *Topics* I and VIII may not be the direct source of inspiration for these scholastic games. (Of course, once scholastic writers started paying attention to *Topics* I and VIII, they were quick to point out the similarities.)

Some modern scholars suggest that obligational games developed organically as one particular kind of disputation. Medieval universities enjoyed a culture of disputation, with a variety of different disputations forming an integral part of the scholastic academic life. It is natural to think that obligational games emerged in this environment as just one particular kind of disputation. See Stump (1989c), Yrjönsuuri (1993), and Dutilh Novaes and Uckelman (2016).

Others suggest that obligational games developed in close connection with medieval interests in counterpossible reasoning, liar paradoxes, *ex falso quodlibet*, and theories of consequence. See Martin (1992) and Stump (1989d), for example.

It is also worth noting that no written transcripts of these debates survive. This has led some to think that these games were never actually played. Instead, they were imagined exercises, carried out in the minds of medieval logicians (see the introduction to Spade 1977; and Lagerlund and Olsson 2001). However, as Sinkler (1992) points out, later authors like Paul of Venice speak about obligational games as if they did happen (for instance, Paul mentions that they often get disrupted), and this suggests that these games actually were played in the medieval university.

Interpreting the Games

The point or purpose of these games is not clear. Both scholastic writers and modern scholars offer a variety of different interpretations.

Scholastic Interpretations

Scholastic writers themselves offer a number of different reasons for the games. Some say the games are meant as classroom exercises designed to help train students in logic (for example, De Rijk 1975, Paul of Venice 1988, Braakhuis 1998). Alternatively, some authors say these games are meant to be a special technique that can be used to solve sophisms (for instance, De Rijk 1976).

Still others say these games are meant as a kind of tool for proof search: they help the players find logical consequences that follow from some proposed thesis (see De Rijk 1974, 1975, Braakhuis 1998). In fact, some medieval authors even claim that these games are useful for (say) lawyers and ethicists, when they want to figure out what follows when something is imagined to be the case (Kretzmann and Stump 1985).

Modern Single-Agent Interpretations

Modern scholars offer a number of interpretations too. Some propose that these games are primitive axiomatic theories (Boehner 1952). Others follow scholastic writers and argue that these games really are just exercises for logic training (Weisheipl 1966; Perreiah 1982; Sinkler 1992; Ashworth 1993), tools for exploring consequences (Yrjönsuuri 2015), or that they are really meant as a tool for exploring sophisms (Stump 1989d).

Martin (2001) suggests that obligational games aim to evaluate whether a set of propositions can be held together, and the way that a respondent adds more propositions as they proceed in a game is reminiscent of constructing a (partial) possible world or model. Yrjönsuuri (1998) makes a similar suggestion, arguing that a respondent constructs a set of propositions in a game which are logically consistent in a syntactic sense. Alternatively, Yrjönsuuri (1996) claims that obligational games can be seen as thought experiments, where the players construct the experiment world as they play the game (see also King 1991).

An oft-cited but controversial interpretation comes from Spade (1982a), who argues that the *positio* game is a theory of counterfactual reasoning. The notion of counterfactual reasoning that Spade has in mind is this: imagine a counterfactual situation in a world that is otherwise as similar to our own as possible, and then see what follows in that world. In such a counterfactual world, anything that logically follows from the counterfactual situation will hold, but everything else will stay the same as in our world.

Spade sees an analogue in *positio* games. By starting with a context, players begin with a world that is much like our own. Then, they propose a counterfactual statement and grant anything that follows from it. Anything that does not follow from that counterfactual statement stays the same as in our world, through the context.

Stump (1989d) argues that this cannot be right. As we have seen, when the players turn to the context to handle irrelevant propositions, players are *not* in fact obliged to concede things that follow from the thesis. On the contrary, players ignore everything they have conceded, and they answer purely from the context. So this does not look like counterfactual reasoning after all (see also Uckelman 2015a).

Another interpretation from Lagerlund and Olsson (2001) is that Burley's *positio* game can be seen as a theory of belief revision. According to the so-called AGM theory of belief revision (Alchourrón, Gärdenfors, and Makinson 1985), belief revision can be modeled as follows. An agent has a belief set that they are committed to (this includes the beliefs the agent knowingly holds plus all other beliefs that follow logically). Beliefs can then be added or removed from the set. Adding or removing any one belief can cause other beliefs in the set to be added or removed too, in order to keep the belief set consistent and to account for any new things the agent would be logically committed to believing (or not).

Lagerlund and Olsson argue that, in a very similar way, Burley saw his *positio* game as a mechanism for updating an agent's belief set with a new belief. The context is the agent's original belief set, the thesis is the new belief to be added, and the game itself is the procedure that one follows to discover additional new beliefs that should be added to the original belief set, because they follow logically.

Dutilh Novaes (2007) questions this approach. Under the AGM model, updating a belief set with a new belief may involve erasing other beliefs in the original set (e.g., beliefs that are not compatible with the new belief). But this cannot be done across the board in obligational games. Under Burley's *positio* rules, for example, a respondent cannot revise **C**. So an obligational game does not look like a mechanism for revising a belief set after all.

Modern Multi-Agent Interpretations

Dutilh Novaes points out that all of the aforementioned interpretations ignore the game-like character of obligational games. In obligational games, there are two players who follow rules to try and beat each other, and this should be accounted for in any interpretation scholars might want to propose. For Dutilh Novaes, obligational games should be seen as genuine games, in the sense that they are multi-agent interactions governed by rules (see also Uckelman 2012, 2013).

For her part, Dutilh Novaes models an obligational game as a quadruple of the following four pieces: (1) the context (the players' beliefs), (2) the propositions the opponent proposes, (3) the propositions the respondent concedes during the game, and (4) a function that assigns to each proposed proposition a correct response. Such a model grows as a game is played, as new propositions are proposed and conceded.

Dutilh Novaes uses this quadruple to model the games of Burley, Swyneshead, and Ralph Strode (with some modifications to the model to account for these different authors' views—see Dutilh Novaes 2005, 2006a, 2006b). In addition, Dutilh Novaes is able to prove that these games have certain properties. For example, under Burley's rules, the respondent can always win, but the game is hard, and it is dynamic (Dutilh Novaes 2005). Under Swyneshead's rules, the games have different properties. For example, they are not dynamic (Dutilh Novaes 2006a).

Uckelman (2011a) also insists that obligational games are interactive and game-like. But she argues that other models of these games (like the ones proposed by Dutilh Novaes) are additive, in the sense that they start with a small set of propositions (e.g., the thesis), and they grow as more rounds are added. By contrast, Uckelman wants to capture the opposite sort of progression, since as a respondent concedes more and more propositions during a game, the set of further propositions they can concede gets smaller.

To this end, Uckelman uses a multi-agent variant of dynamic epistemic logic (e.g., van Ditmarsch, van der Hoek, and Kooi 2008) to model the games. The model begins as a large model that represents the comprehensive knowledge of the players about the world. The actions of the players are then encoded as model-reduction operations: for instance, when the respondent concedes a new proposition, all pieces of the model that are no longer compatible are cut out.

One advantage of Uckelman's approach is that she can model a variety of different properties of obligational games. In addition, she can adopt her general-purpose model to handle other types of games (see Uckelman 2013). One interesting case is *dubitatio*, where the respondent aims to keep a particular proposition doubtful (Uckelman 2011b).

Conclusion

Fifty years ago, there was little modern scholarship on these games. Today, there is a growing body of literature, of which I have only scratched the surface. In addition to the references mentioned already, there are a number of introductory surveys on the topic, e.g., Stump (1982), Spade (1982b), Yrjönsuuri (2001b), Dutilh Novaes (2008), Uckelman (2012), and Dutilh Novaes and Uckelman (2016). There are a few book-length studies, e.g., Yrjönsuuri (1994), Keffer (2001), and Dutilh Novaes (2007). There is an extensive bibliography up to 1994 in Ashworth (1994), and up to 2015 in Uckelman (2015b).

Nevertheless, a great deal more research remains to be done. For example, modern scholars have concentrated mostly on the *positio* game, but what about the other types of games? There are only a handful of studies on the other kinds (for example, Uckelman 2011b, 2015a), so more can be done in that respect. Moreover, each time modern scholars come up with a new mathematical model for these games, new insights are generated. What other kinds of models may help us understand these games? Finally, our understanding of the history and development of these games (even into the late medieval period) is still somewhat spotty, and the question of how to interpret these games is anything but settled.

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