

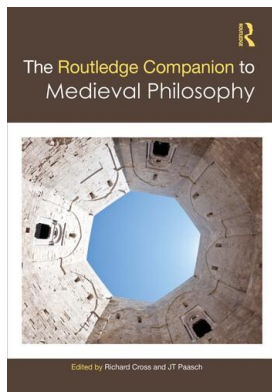
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4

MODAL LOGIC

Spencer C. Johnston

The aim of this chapter is to introduce some of the main themes in medieval discussions of modal logic. In particular, this chapter will provide a high-level overview of a number of different aspects of the medieval discussions around modal logic before focusing on one specific account of modal logic, that of John Buridan, and its relationship to Aristotle's modal logic. To do this, I will first outline some of the necessary background, logical terminology, and ideas that are presupposed by the medievals' own analysis.¹ I will then discuss three accounts of the modal operators that can be found in medieval texts: a temporal interpretation of modal expressions, an essentialist interpretation of modal expressions, and a possibility-as-alternative account of modal expressions. I will go on to illustrate how these ideas come together into a complete analysis of modal expressions by looking at how one account (that of the fourteenth-century arts master John Buridan) was developed and comparing it to some of the other accounts of modal logic that one finds in the medieval period.

It was standard in medieval discussions of modality to single out six alethic modal expressions for specific consideration: necessity, possibility, impossibility, contingency, truth, and falsity.² In this chapter, I will limit my attention to the first four modal expressions.³ The reason for this restriction is primarily one of space. A full treatment of truth and falsity would inevitably require a discussion of the medieval *sophismata* literature in general, and specifically medieval discussions about theories of truth and logical paradoxes, which is well beyond the scope of the present chapter.

My main focus in this chapter will be to look at medieval logic in the scholastic tradition in the high middle ages. Though I restrict myself primarily to the Latin tradition, this is not to suggest that there was not very important and interesting work being done on modal logic in the Arabic or Greek world. For more on Arabic modal logic, see Dutilh Novaes and Read (2016: chs. 2, 3, and 14).

Background and Motivations

The background for medieval discussions about modality in general, and about modal logic particularly, comes from a variety of sources. One important set of sources are the writings of Aristotle, first as translated by Boethius and others, and then obtained through interactions with the Arabic speaking world. Another important source of modal ideas comes through the various theological writings of the early Christian church. As we shall see, a number of the ideas of, for

example, Augustine, come to have important implications for how modality is conceived of in the late thirteenth and fourteenth centuries.

For the more specific sources dealing with logic, the main ancient works that served as the background for medieval logic come primarily from Boethius's translations of Aristotle's *Categories*, *On Interpretation*, and Porphyry's *Isagoge*; Boethius' commentaries on these works and on Cicero's *Topica*; and Aristotle's own logical writings (Knuuttila 2008: 505).

The Latin texts that comment on and work from these sources are sometimes referred to as comprising the *logica vetus*, i.e., the old logic. As Latin authors engaged with the remaining works of Aristotle's *Organon* (the *Prior Analytics*, *Topics*, *On Sophistical Refutations*, and the *Posterior Analytics*) further questions about logic arose. In the context of modal logic, the *Prior Analytics* deserves particular mention, since it includes Aristotle's theory of the syllogism. This discussion raised a number of questions about both the correct interpretation of his theory and its underlying plausibility.

As Latin authors developed their own distinct semantic and logical theories, they entered into the period of the *logica modernorum*, i.e., the logic of the moderns. This is generally considered to have started in the twelfth century, and to have reached its zenith in the fourteenth century.

When one looks at the texts in which medieval discussions of modal logic occur, it becomes clear that these texts were intended to serve a number of different purposes, including being commentaries on the texts of Aristotle and others, textbooks, and independent treatises. These categories are not mutually exclusive, as for instance, Buridan's *Summulae Dialectica* was both a textbook and a commentary on Peter of Spain's *Summulae Logicales*.

Background Context

From the above-mentioned sources, medieval authors inherited a set of background ideas and terminology that would become standard for their discussions of logic (Dutilh Novaes and Read 2016: 290). For medieval authors, a syntactically categorical proposition can be described as having the following form:⁴

Quantifier, Subject, Copula, Predicate

The copula can take a variety of forms, but for the moment, we will consider only "is" and "is not," which are called assertoric copulas. Later we will consider how the copula can be modified in modal contexts.

The subject and the predicate were both called the terms of the proposition. Terms could be simple, e.g., "human," "animal," or "white," or they could be complex, where one of the terms is, for example, itself a categorical proposition or a relational expression. Indeed, in some cases (e.g., propositions with oblique terms), it was not always clear which part of a proposition counts as the subject and which part counts as the predicate (Buridan 2015: 128–129).⁵ However, for the purposes of this chapter, we can limit ourselves to cases where the terms in question are either simple or are composed of one term that is in an accusative-infinitive construction.

These propositions are used to state when the subject belongs to "all," "no," "some," or "not all" of the predicate. These are the four possible forms that the quantifier can take in such a proposition. If the quantifier is either "all" or "no" the proposition is said to be universal. If the proposition is either "some" or "not all" the proposition is said to be particular. If the proposition affirms that the subject does hold of the predicate (i.e., it holds for "all" or "some"), then the quantity of the proposition is said to be affirmative. Likewise, if the proposition denies that the subject holds of the predicate, then it is said to be negative.

Square of Opposition

The square of opposition is a diagrammatic way of representing a number of inferential relationships that hold between categorical propositions.

<i>Every A is B</i>	∇	<i>No A is B</i>
↓	X	↓
<i>Some A is B</i>	\leftrightarrow	<i>Not every A is B</i>

Here we are using the symbol ∇ for contrariety, \leftrightarrow for subcontrariety, ↓ for the relationship of subalternation, and X for contradiction, which should be read as two lines, one from “Every A is B” to “Not every A is B,” and another from “No A is B” to “Some A is B.”

According to Peter of Spain, the relationships of contrariety, subcontrariety, contradiction, and sub-alternation can be defined as follows (2014: 113).

Two propositions, A and B, are contrary if A’s being true entails that B is false.

Two propositions, A and B, are sub-contrary if A’s being false entails that B is true.

Two propositions, A and B, are contradictory if A is false entails that B is true, and that if A is true, then B is false.

B is subcontrary to A if A entails B and not vice versa.

Syllogisms

An important part of medieval logic focused on the validity of syllogisms. The definition of a syllogism goes back to Aristotle, where he states that:

A deduction is a discourse in which, certain things being stated, something other than what is stated follows of necessity from their being so.

(1985: 40, 24b18)

The word “syllogism” itself is a Latinization of the Greek term *sylogismos*, which is here translated as “deduction.” When a proposition does not contain a modal expression or a temporal modifier, it is called an assertoric proposition, and likewise, when a syllogism contains only assertoric propositions, it is called an assertoric syllogism. The medieval understanding of this definition and how to properly interpret it led to a number of interesting and important discussions that are outside the scope of the present chapter.⁶

For our purposes, in this chapter we will use Buridan’s definition of syllogism as the point of departure for our discussion of modal syllogisms. In the *Treatise on Consequences*, he says:

So we want to understand by “syllogism” in what follows only a formal consequence to a single subject–predicate conclusion by a middle [term] different from each of the extremes in the conclusion.

(2015: 115)

The account that Buridan is developing here is situated within a more general framework of logical consequences. In the first book of his *Treatise on Consequences* Buridan had given a general theory and definition of when a consequence is valid and then divided consequences into those

that are formally valid and those that are materially valid. Syllogisms, Buridan is saying here, are a type of formal consequence.

Syllogisms can be organized according to the position in which the subject, middle, and predicate term occur. Using S for the subject term, M for the middle term, and P for the predicate term, we obtain the following three figures:

	<i>First figure</i>	<i>Second figure</i>	<i>Third figure</i>
First premise	M × P	P × M	M × P
Second premise	S × M	S × M	M × S
Conclusion	S × P	S × P	S × P

As many medieval authors observe, there is a fourth figure that is also possible, namely:

	<i>Fourth figure</i>
First Premise	P × M
Second Premise	M × S
Conclusion	S × P

Buridan differs from authors who affirm a distinct fourth figure, however:

But it should be noted that the fourth figure differs from the first only in the transposition of the premises, and that transposition does not permit inferring another conclusion or prevent that inference, but affects whether the conclusion inferred is direct only when in the first figure and indirect in the fourth and vice versa . . . From this it is clear that once the first figure has been explained . . . it will be superfluous to explain the fourth; so Aristotle does not mention it.

(2015: 116)

That is to say, once we notice that the fourth figure relates to the first figure in this way, there is no need for subsequent discussion of it.

The medieval writers on logic used a helpful set of mnemonic devices to aid in the memory of the various valid syllogisms and to encode information about how second and third figure syllogisms are to be reduced to first figure syllogisms.⁷

The mnemonic (here quoted from Peter of Spain 2014: 191) is as follows:

Barbara Celarent Darii Ferio Baralip-ton
 Celantes Dabitis Fapesmo Frisesomorum
 Cesare Cambestres Festino Barocho Darapti
 Felapto Disamis Datisi Bocardo Ferison

Each of the names corresponds to one of the valid syllogism schemata that Aristotle identified under each figure. So, Barbara is the first valid schema under the first figure, and so on. Each of the vowels in these names—“a,” “e,” “i,” and “o”—corresponds to one of the categorical propositions discussed above. Here “a” is a universal affirmative, “e” the universal negative, “i” the

particular affirmative, and “o” the particular negative. As such, the name Barbara is used to pick out the syllogism:

Every B is C
Every A is B
Therefore, every A is C

While Datisi is the syllogism:⁸

Every B is C
Some B is A
Therefore, some A is C

These names are still used in contemporary discussions about syllogisms.

Medieval Modal Logic Background

Having provided the necessary background about assertoric propositions and syllogisms, I can now go on to discuss medieval modal logic.

Syntactically, modal propositions can be obtained by adding a modal expression to an assertoric proposition. This can be done in two ways. The first way is to allow for the copula to be modified by a modal adverb. For example, “is possibly,” “is necessarily,” “is not necessarily,” etc. This allows for the creation of propositions such as “Every human is necessarily an animal” and “Every donkey is not necessarily white.” Some medieval authors refer to this as a divided modal proposition (e.g., Buridan 2015: 95–96).

The other way to obtain modal propositions is to allow for one of the terms in a proposition to be a modal expression. In Latin this was normally done using an accusative-and-infinitive construction, where the modal term is nominative, and what is being modified by the modality is in an accusative-infinitive construction. In practice the copulae in these propositions are usually assertoric. This preserves the structure of the categorical proposition as given above. However, in English translations, such constructions are generally translated using “that” clauses. An example of such a proposition would be “It is necessary that some man is an animal.” Such propositions are sometimes referred to as composite modal propositions. According to Read:

The distinction between a term being divided or composite finds its origins in Aristotle’s *Sophistical Refutations* (see 166a23–38) and was an extremely important distinction from Peter Abelard onward. The distinction is similar to the modern distinction between a term being read *de re* or *de dicto*, although the medieval distinction is broader in its use.

(Buridan 2015: 30)

Turning to syllogisms, a modal syllogism is a syllogism where at least one of the propositions is modal. In some parts of the modern literature the syllogistic mnemonic is extended by the use of the letters L, M, Q, and X, which stand respectively for necessity, possibility, contingency, and assertoric (i.e. no modality present). In such cases, the three letters are usually placed before the syllogism, so LXL Barbara would be the syllogism:

Every B is necessarily C
Every A is B
Therefore, every A is necessarily C

While XLL Barbara would be:

Every B is C
Every A is necessarily B
Therefore, every A is necessarily C

These two syllogisms are of particular importance, as Aristotle famously claimed that the first is valid, while the second is not (see *Prior Analytics* 30a15–30b6). It has often been noted in modern discussions of Aristotle (e.g., McCall 1963: 10–15) just how difficult it is to understand Aristotle’s modal logic.⁹ This difficulty was also appreciated by medieval logicians reading his texts. In general, the systems developed by medieval authors did not match the list of validities and invalidities given by Aristotle, nor did they agree with each other. Unlike the assertoric syllogism, where which syllogisms were valid was a settled issue, the question of what modal syllogisms were valid varied considerably. It will be helpful to look at aspects that contribute to this difference, the various ways medieval authors understood the modal terms, and the ways that medieval authors understood the causes of truth for the various propositions.

Interpreting the Modal Operators

Among medieval authors, one can discern a number of different approaches to thinking about modality in general, and about modal logic in particular. These issues of interpretation sometimes intertwine with the question of which modal syllogisms are accepted or rejected by particular authors. I will focus on three medieval approaches to interpreting the modal operators—namely, temporally, essentially, and with reference to God as securing alternative possibilities. For further discussion of other modal interpretations, see Knuttila (2008: 505–530).

Temporal Modals

One view about modality that the medievals inherited is the idea that the operations of possibility and necessity should be understood with respect to time. In the classical tradition this issue goes back (at least) to a set of disagreements about modality between the Stoics and the Megarians (see Rescher and Urquhart 1971: 4).

The Stoic idea had been to identify the actual with the things that are currently realized now. For example, “Socrates is actually running” is true just in case Socrates is (at this time) running. Possibility, on this view, is then glossed in terms of being true at some point in the future. That is, “It is possible that p ” should be glossed as saying “There is some time in the future such that p ” (see Rescher and Urquhart 1971: 4). Necessity, on this view, is glossed in terms of all times in the future. That is, “It is necessary that p ” is true if and only if, for all times in the future, p holds.

When we come to the medieval employment of this notion, one of the most influential examples of this analysis is found in William of Sherwood’s *Introductiones in logicam* (translated in 1966). Sherwood’s textbook, along with Peter of Spain’s *Summulae*, and (to a lesser degree) Lambert’s *Logica* are important logical manuals from the thirteenth century.

A helpful example of the temporal modal interpretation is found in Sherwood’s discussion of how the term “impossible” can be used. According to Sherwood:

Notice, however, that “impossible” is used in two ways. It is used in one way of whatever cannot be true now or in the future or in the past (*quod non potest nec poterit nec potuit esse verum*); and this “is impossible *per se*” – e.g., “A man is an ass.” It is used in the other way of

whatever cannot be true now or in the future although it could have been true in the past, as if I were to say, I have not walked; and this is impossible *per accidens*.

(1966: 41)

Sherwood then goes on to make the point that a similar analysis applies in the case of the terms “necessity,” “possibility,” and “contingency.” The key point to observe here is that, according to Sherwood, both of the ways to use these modal expressions are connected to time. In particular, we have the *per se* reading:

Impossibly p if and only if p cannot be true now, or in the future, or in the past.

Necessarily p if and only if p is true now, in the future, and in the past.

Possibly p if and only if p is either true now, or in the future, or in the past.

And the *per accidens* reading:

Impossibly p if and only if p is not true now or in the future.

Necessarily p if and only if p is true now and true in the future.

Possibly p if and only if p is either true now or in the future.

The main difference between the two possible ways of reading each of the modal expressions is due to whether the modality is only “forward looking” as in the case of the *per accidens* reading, or if it also includes the past.

Essentialist Modals

A second interpretive tradition, exemplified by the writings of Peter Abelard and Robert Kilwardby, analyzes modalities in terms of the essences or natures of the things under consideration. These essentialist considerations are part of a broader metaphysical framework with roots going back to Aristotle. For our purposes here, it will be sufficient to think of an essence as a kind of “real definition.” The idea comes from Aristotle’s remarks in *Topics* 102a3, where he writes, “A definition is a phrase signifying a thing’s essence” (1984: 169). The idea is then to define necessity in terms of that which is part of the essence of the thing (i.e., part of its real definition), and possibility in terms of that which is not in conflict (i.e., not incompatible) with a thing’s essence.

According to Abelard, the right way to think about a modal proposition, such as “It is necessary that Socrates is a human,” is to think of it in terms of the essential relationships that hold between the terms. In the case of a necessary proposition, the predicate needs to be required by the subject. So, in the case of “Socrates” and “human,” if we look at Socrates’ essence, we will find that human is one of the terms that occurs in the definition, and, as such, being human is part of Socrates’ essence. Hence the necessary proposition is true. In a similar spirit, a proposition of possibility is true if it expresses a relationship that is not repugnant with the essence of the thing. For example, the proposition “It is possible that Socrates is running” is true if running is not incompatible with Socrates’ essence.

It should be observed that, on this reading, essence need not reduce to temporal instances, since it may turn out that things that are possible on this view never occur at any point in time. Abelard illustrates this with the example of Socrates being a bishop. The basic idea is that, for it to be possible that Socrates be a bishop, it does not mean that Socrates must, at some point in time, have been, be, or become a bishop (*contra* the temporal interpretation we discussed previously). Instead, all that is required is that becoming a bishop not be incompatible with the essence that

Socrates has. This illustrates one of the important differences between the two interpretations of the modal expressions.

A similar analysis is given by Kilwardby in his commentary on the *Prior Analytics*. The aim of this work was an exposition and elaboration of the *Prior Analytics*, and so it contains an important analysis of modal propositions. According to Thom:

Kilwardby's basic idea is that in order for a proposition to be necessary, it is not enough that it be true and be incapable of not being true; rather, the proposition has to state an essential and inseparable cause of the predicate inherence or non inherence in the subject (2016; *Lectio* 40: 162). By "an essential and inseparable cause of inherence," Kilwardby understands the *per se* predications mentioned at *Posterior Analytics* I.4, 73a35ff (Kilwardby 2016; *Lectio* 9: 458). In the case of affirmative propositions, the necessary is that which is contained in the what-it-is, the essence, of the subject, as a part or an essential difference of that essence. In the case of negatives, it is what is excluded by the essence of the subject. In either case, necessity is grounded in the Aristotelian notion of essence.

(2016: 361)

Modalities as Alternatives

The final way to interpret modalities that we will consider in this chapter is the idea of modality as expressing what is within the power of God to bring about. This analysis is closely associated with John Duns Scotus, although its theological origins go back at least to Augustine, if not to the biblical authors (Knuuttila 2008: 517).

The starting place for this conception of modality is Augustine's account of creation. The relevant parts for our discussion are summarized by Knuuttila as follows:

Augustine argued that God simultaneously created the first things and the seminal reasons for later things out of nothing. The creation was based on an eternal free act of God's perfectly good will, and took place through his omnipotence. In Augustine's Trinitarian view, the Son is a perfect image of the Father and, as the Word, the seat of the ideas of finite beings which in a less perfect manner can imitate the highest being. The ideas refer to possible actualization in the domain of mutability. In this sense the possibilities have an ontological foundation in God's essence. This became the dominating conception of theological modal metaphysics until Duns Scotus departed from it.

On Augustine's view, the act of creation is an action that God did bring about, but was not required or necessitated to bring about. Likewise, the world, on this view, is not necessary, but contingent. What developed, on subsequent reflection, was the idea that modality could be seen as a choice between different, alternative ways things could be brought about. For example, it was within the power of God to create the world, but it was also within the power of God to refrain from creating the world.

(2008: 517)

In its simplest form, this view would become:

It is possible that p just in case it is within the power of God to bring about p .

It is necessary that p just in case it is not within the power of God to bring about not- p .

In the twelfth century, these ideas were further developed, distinguishing between, e.g., natural necessity and unrestricted necessity, which was used to draw out the idea that although the natural world

and its laws are contingent, it does still have a rule-like nature, such that things happen or fail to happen in accordance with how the laws are set up. However, these patterns in nature are not absolute, they are themselves created by God, who can, in principle, overrule such laws. This in turn led into interesting theological discussions about God's relationship to the created world, like questions about the relationship between human choice and the divine will.

Under the influence of Duns Scotus, these ideas were further transformed into the idea that God acts by choosing between different alternatives. According to Scotus:

I do not call something contingent because it is not always or necessarily the case, but because the opposite of it could be actual at the very moment when it occurs.

(Knuuttila 2008: 550, citing Scotus's *Ordinatio* 1.2.1.1–2, 86)

In this definition of contingency, we find a rejection of the traditional thesis of the necessity of the present.¹⁰ What is new in Scotus's analysis was how he formulated the foundations of these modalities. In the Augustinian tradition, the source of modality was to be found in God's essence. In contrast, Scotus argued that God, in his omnipotence, does not turn to his essence in order to understand what is possible, but instead possibilities are known by God in themselves. Indeed, they would still be possible even if God did not exist.¹¹

This led Scotus to draw a number of distinctions in modality. A proposition was called *logically possible* if its description did not involve a contradiction. According to Scotus, God has knowledge of all of these possibilities, and among these possibilities God chooses to actualize some, while not actualizing others. Those that are actualized by God obtain their own being, while the possibilities that are not actualized remain ideas in God's mind. Thus, the world is "put together" out of various compossibilities by God. Two beliefs are compossible if the conjunction of those beliefs is possible. Likewise, impossibility, on Scotus's view, is just impossibility between the various ideas.

Systems of Modal Logic

It will be helpful to conclude this chapter by looking at an actual system of medieval logic, to give a flavor for how these systems function and how they relate to one another. In order to do this, we will start by looking at relationships between the various modal expressions, given by a modal "square of opposition." We will then look at Buridan's single premise inferences and his account of the modal syllogism.

Relationships between Modal Expressions

As we have already discussed, there are four main modals that medieval authors focused on in their treatment of modal logic; namely necessity, possibility, contingency, and impossibility. It would be helpful, then, to look at how medieval authors understand the relationships that existed between these terms, and modifications of these terms with negations. Thirteenth-century logical texts often discuss this in a manner similar to how the square of opposition was illustrated for categorical propositions.

Peter of Spain (2014: 125–126) groups the modal expressions into the following four groups of equivalent propositions:¹²

I	II
Possibile est esse (is possible to be)	Possibile est non esse (is possible to not be)
Contingens est esse (is contingent to be)	Contingens est non esse (is contingent to not be)
Non impossibile est esse (is not impossible to be)	Non impossibile est non esse (is not impossible to not be)
Non necesse est non esse (is not necessary not to be)	Non necesse est esse (is not necessarily to be)

<p>III Non possibile est esse (is not possible to be) Non contingens est esse (is not contingent to be) Impossibile est esse (is impossible to be) Necessesse est non esse (is necessary not to be)</p>	<p>IV Non possibile est non esse (is not possible not to be) Non contingens est non esse (is not contingent not to be) Non impossibile est non esse (is not impossible not to be) Necessesse est esse (is necessary to be)</p>
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As the reader can verify, the first group deals with propositions that are equivalent to ones of possibility, while the fourth group deals with propositions of necessity and their equivalents. Groups II relates propositions that are non-necessary, while group III corresponds to propositions of impossibility.

Peter then goes on to discuss the relationships of contrariety, subcontrariety, contradictory, and sub-alternation between these different groups. According to Peter the relationship looks like the following:

IV	↔	III
↓	X	↓
I	↔	II

Notice that the relationships given here are very similar to those given in the square of opposition. As an illustration of this table, we can tell that a proposition like “Some A is necessarily B” (which belongs to group IV) is contrary to “Some A is necessarily not B,” contradictory to “Some A is not necessarily B,” and superaltern to “Some A is possibly B.”

Singular Inferences

In discussing singular inferences, it will be helpful to narrow our focus to the account given of divided modal propositions by Buridan in his *Treatise on Consequences*. In this text, Buridan focuses on the four modal propositions we have been discussing, although for our purposes here we will focus primarily on divided propositions containing the expressions “necessarily” and “possibly.” Buridan outlines the causes of truth for these different propositions by employing a theory of supposition. Supposition is the relationship that different terms have to objects and it was standard to distinguish three kinds of supposition, namely, personal, material, and simple. A term has personal supposition if it supposits for the things it picks out. The idea behind personal supposition is that “a term, when it occurs in a proposition ‘stands for’ [*supponit pro*] each member of a certain class of things and the truth conditions are stated in terms of the relationship between these classes” (Hughes 1989: 93–94). A term has material supposition if it supposits for the term itself. For example, in “Socrates has 8 letters,” the term “Socrates” has material supposition. A term has simple supposition if it supposits for the universal, as in “Human is a species,” or “Animal is a genus.”¹³ In what follows we will only be making use of personal supposition.

What is notable about Buridan’s account of divided modal propositions is that, on his analysis, the subject term is *ampliated*. The basic idea of ampliation is that the supposition of the term is extended to range over more things than its grammatical structure would initially suggest. In the fourteenth century, this was often employed to treat terms like “dead.” In a sentence like “A is B,” a common fourteenth-century analysis would say that this is true if there is at least one presently existing thing that A and B both supposit for. However, in a sentence like “Socrates is dead,” there is no *presently existing* thing that “Socrates” stands for. The medieval way around this was to posit

that terms like “dead” amplified their subject. In the case of dead, it amplified its subject to the past, so that the conditions for “Socrates is dead” to be true are “Something that is or was Socrates is dead.” This in turn can be true, since we can pick out Socrates in the past, and observe that he no longer exists.

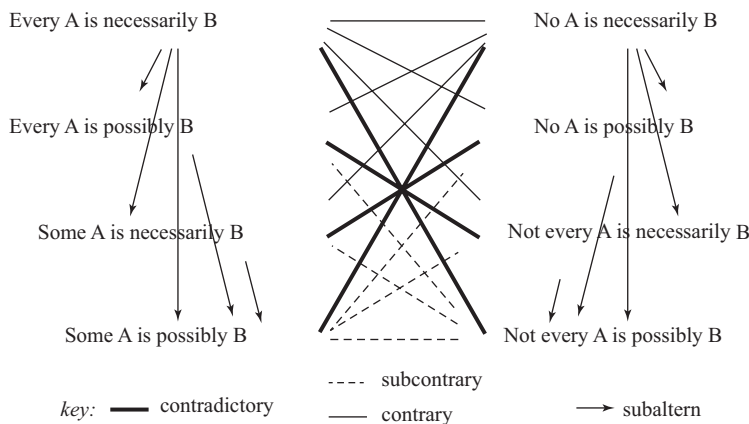
In the case of modal propositions, Buridan requires that the subject of such propositions be amplified not to what is past, but to what is possible. As such, Buridan gives them the following analysis:

Proposition	Conditions
Every A is possibly B	Everything that is or is possibly A is possibly B
Some A is possibly B	Something that is or is possibly A is possibly B
No A is possibly B	Nothing that is or is possibly A is possibly B
Not every A is possibly B	Not everything that is or is possibly A is possibly B
Every A is necessarily B	Everything that is or is possibly A is necessarily B
Some A is necessarily B	Something that is or is possibly A is necessarily B
No A is necessarily B	Nothing that is or is possibly A is necessarily B
Not every A is necessarily B	Not everything that is or is possibly A is necessarily B

In each case, while there is no explicit modality included in the proposition, the subject is amplified to the possible. In fact, the table can be further simplified by observing that in the subject the disjunction “that is or is possibly” can be simplified to “is possibly,” since Buridan accepts the inference “if p then possibly p .” Here we can also see one place where Buridan differs from other authors. In particular, the choice to have the subject of a necessity proposition be ampliative is a rejection of the analysis that Ockham gave in his *Summa*.

Modal Octagon

As before, we can visually represent the relationship between the various propositions that Buridan treats. In this case, we need to extend the square of opposition into an octagon of opposition.



Modal Syllogism

In his *Treatise on Consequences* Buridan’s analysis of the modal syllogism comes at the end of the work. The *Treatise* is divided into four books, with the first treating consequences between propositions

generally, the second single premise inferences between modal propositions, the third treating syllogisms (the third book is further divided into two parts, the first treating categorical syllogisms with simple terms, the second treating syllogisms with oblique terms), and the fourth and final book providing a systematic discussion of modal syllogisms, ending with some remarks about reduplicative propositions. Buridan’s account of the modal syllogism, and the syllogism in general, is very much centered within his broader discussion of consequences. For example, Buridan defines a syllogism as a formal consequence that has a particular kind of form (2015: 115–116). A full description of the validities in Buridan’s modal system is documented by Read in (2015: 41).

For our comparisons here, we will focus only on the validities of first figure syllogisms, reproducing below a partial version of Read’s table. The modality in each of the columns corresponds to the major premise in the argument, while the modality in each of the rows gives the modality of the minor premise. For example, the second row with the first column gives the modality of the premises of a major of necessity and a minor of possibility. Note that Buridan only considers modal syllogisms that have an underlying assertoric form that is valid, e.g., Barbara or Darapti. When only a modality is listed below, this means that all of the assertoric syllogisms are valid with this modal combination. If only some syllogisms are valid, then the modality will be given first, and then the name of valid syllogism given. A pattern with no validities will be left blank.

Buridan’s valid first figure modal syllogisms

	<i>Necessity</i>	<i>Possibility</i>	<i>Assertoric</i>	<i>Contingency</i>
<i>Necessity</i>	Necessity	Necessity, Possibility, Assertoric – Celarent	Necessity – Darii, Ferio Assertoric – Barbara, Celarent	Necessity, Possibility, Assertoric – Celarent
<i>Possibility</i>	Possibility	Possibility	Possibility – Darii, Ferio	Possibility
<i>Assertoric</i>	Possibility, Assertoric – Celarent		Assertoric, Possibility – Darii, Ferio	
<i>Contingency</i>	Possibility, Contingency	Possibility, Contingency	Contingency – Darii, Ferio	Contingency

It is interesting to observe that Buridan differs from Aristotle on the validity of a number of syllogisms. Focusing just on the necessity and assertoric components will illustrate this. According to Aristotle the following syllogisms are valid (the table is adapted from Malink 2006: 117).

Aristotle’s valid first figure modal syllogisms

	<i>Necessity</i>	<i>Assertoric</i>
<i>Necessity</i>	Necessity	Necessity
<i>Assertoric</i>		Assertoric

Notice here that Buridan’s system is weaker than Aristotle’s with respect to the Necessity–Assertoric moods. While Aristotle accepts that validity of all the Necessity–Assertoric–Necessity syllogisms (see *APr.* 30a17–30b1), Buridan only accepts the validity of Necessity–Assertoric–Necessity Darii and Ferio (i.e., only the syllogisms that conclude to a particular conclusion, not to a universal). However, Buridan does allow for Necessity–Assertoric Barbara and Celarent to be valid, just

not to a conclusion of necessity, but only to one that is assertoric. Given what has been said, one might be tempted to think that Buridan's reasons for doing this is to keep his treatment of the modal syllogism as close to Aristotle's as possible while retaining the analysis of modal proposition that Buridan argues for in his *Treatise on Consequences*. However, there is no suggestion of this in the text of the *Treatise*. For example, the conclusion where Buridan treats Necessity–Assertoric–Necessity Barbara and Celarent is extremely brief and contains no references to Aristotle at all.

Conclusion

Our aim in this chapter has been to introduce basic ideas of medieval modal logic. To that end we have looked at three different ways that medieval authors thought about modal expressions and then looked at how medieval logic treated syllogistic arguments with modal terms. As this brief and incomplete survey has hopefully shown, there was a considerable plurality of views about the right account of modal logic, and disagreements about how to understand the modal operators. However, there is much in the medieval literature on this topic that has not been discussed. Our focus has been only on four modal expressions. We provided no discussion of medieval theories of truth and falsity. Likewise, while six modal expressions were the regular focus of medieval authors, they were well aware of other kinds of modalities, e.g., epistemic or deontic modals. See Boh (1993) for discussion of epistemic modal expressions, and Uckelman (2009) for Anselm's logic of agency. The discussion here has been almost entirely "pure" in the sense that it focused only on the accounts of modal logic themselves, not what kinds of arguments someone might want to make using these kinds of valid modal arguments. For example, absent above is any discussion of how various modal notions were employed or made use of outside of logic. In particular, theological discussions about (say) divine foreknowledge and freedom, or what is within the power of God to bring about, are deeply connected to the interpretations discussed here, but we did not delve into them here. Likewise, one finds discussions of the function of modality in medieval texts dealing with themes like fortune and chance, or about the creation of the world. Again, discussion of this has been omitted here, for reasons of space. Nevertheless, the topics presented in this chapter will hopefully enable the reader to begin to situate themselves in the world of medieval modal logic and to find the issues or topics that are of interest to them.

Notes

- 1 In doing this, a number of simplifications are necessary. In particular, the presentation of the syllogism in this chapter standardizes and simplifies the presentation of the syllogism, and does not reflect the questions regarding how a syllogism should be defined.
- 2 See, for example, William of Sherwood (1966: 40); Peter of Spain (2014: 119), and Buridan (2015: 95).
- 3 In what follows we will use the term "modal expression" in a broad sense to include the presence of a modal, regardless of the grammatical form it takes in the proposition. This also includes cases where one of the modal expressions is not explicitly present in the expression, but for instance the verb has modal force (e.g., verbs like "can" or "must").
- 4 The order of the subject and predicate given here is the one used by, e.g., Buridan. In the *Prior Analytics*, the placement of the quantifier and the order of the subject and predicate are reversed, so that, e.g., "Every A is B" would be stated as "B belongs to every A." See for example *APr.* 25a14–25.
- 5 As an illustration of this, Buridan uses the example of the proposition "Every bishop's ass is running" (2015: 128).
- 6 As a starting point, the interested reader should consult Dutilh Novaes and Read (2016: ch. 12).
- 7 For more on this, see Peter of Spain (2014: 191).
- 8 In his explanation for the verse, Peter of Spain says that the last word of the third line, and the fourth line, are syllogisms in the third figure. By this we know that *Datisi* is a third figure syllogism.
- 9 However, see Malink (2006, 2013).
- 10 One standard expression of this is: "what is, necessarily is when it is."

- 11 See Dutilh Novaes and Read (2016: fn 167).
- 12 Compare with Aristotle's remarks in *De Interp.* 13, 22a21–31.
- 13 While this was common, Buridan, because of his nominalist metaphysics, argued that simple supposition was an unnecessary distinction.

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