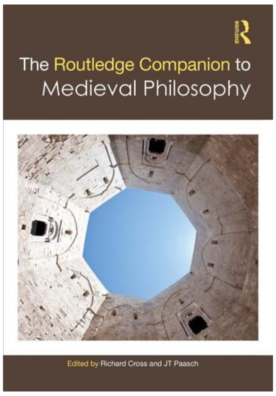

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ATOMISM

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It is a commonplace to assert that atomism, the theory according to which reality is ultimately composed of indivisible entities, disappeared during the Middle Ages. This would be the result of a lack of knowledge relative to Ancient atomism before the rediscovery of Lucretius' *On the Nature of Things* by the Italian humanist Poggio Bracciolini in 1417 and Ambroggio Traversari's Latin translation of Diogenes of Laertius' *Lives of the Philosophers* in 1433, which contains Epicurus' *Letter to Herodotus* (Greenblatt 2011). These translations undoubtedly had a huge impact on Early Modern philosophy, as far as they lastingly changed our conception of the natural world (Wilson 2008). Nevertheless, medieval thinkers did have access to the atomist physics of Leucippus and Democritus through Aristotle's critics (from the twelfth century onward in the West and even earlier in the Arabic and Jewish traditions). Moreover, quite early, Western philosophers were aware of Epicurus and Lucretius' teaching through many intermediate sources, such as Cicero, Seneca, Lactantius, or Jerome to mention only a few (Jones 1989). What is more, Lucretius' poem *On the Nature of Things* never ceased to be copied and read during the Middle Ages, at least for the teaching of Latin grammar, and our modern editions are still based on these medieval manuscripts (Butterfield 2013).

Notwithstanding this substantial knowledge of the Democritean and Epicurean traditions, the fact is, however, that medieval atomists have usually little in common with their illustrious ancestors. As Andrew Pyle (1995) suggested, atomism can be defined by four central claims: (1) a commitment to indivisibles; (2) a belief in the existence of *vacuum*; (3) reductionism (every kind of motion can be explained by the motion of atoms); and (4) mechanism (or reduction of causality to efficient causes). In the Middle Ages, most of the atomists did not agree with (2)–(4). They were not materialists at all, and most often they adopted a mixed view combining atomism with Aristotelian hylomorphism. Likewise, they did not accept the basic tenets of Democritus and Epicurus' systems, such as the plurality of the worlds, the mortality of the soul, the negation of divine providence, or hedonism. So, even if their arguments for the existence of indivisible constituents (claim 1) are sometimes reminiscent of Ancient atomism, their aim was radically different.

Let us mention, for instance, the ninth- and tenth-century Arabic theologians of the *Kalâm*, such as Abū al-Hudhayl and al-Nazzām, who introduced the concept of atom as a central piece of their physics and metaphysics in order to defend occasionalism and determinism (Dhanani 1994; Pines 1997). According to them, God re-creates the combinations of atoms at each and every instant of time, so that every being is totally dependent on his action. This is obviously and totally in contradiction with the Epicurean negation of divine providence. In the same way, the debates in the Jewish tradition, from Maimonides to Hasdai Crescas, are for the most part a reaction to this radical form of atomism (Rudavsky 2000).

The situation in the Latin West is quite different. The supporters of atomism were also influenced by theological considerations about divine creation, the eternity of the world, or the infinity of God, but occasionalism and determinism were not their concern anymore. When, in the twelfth century, Peter Abelard and William of Conches explained that bodies, portions of space, or spans of time are ultimately composed of indivisibles (*indivisibilia* was the Latin equivalent of the Greek *atomoi*), i.e. points, indivisible places, and instants, they mainly referred to Plato's geometric atomism as developed in the *Timaeus* and to Boethius' paraphrase of Nicomachus of Gerasa's *On Arithmetic*, a Pythagorean treatise on numbers (Pabst 1994). They primarily endeavored to describe the creation of the world in terms of basic units in a coherent way with Aristotle's analysis of quantity in the *Categories*. Following this line of thought, they tended to equate the arithmetic units of the Pythagorean tradition with Plato's geometric atomism (Albertson 2014). While Plato affirmed that bodies are made of elementary geometric plane surfaces, the twelfth-century atomists considered these geometric figures to be ultimately constituted of indivisible points. Consequently, they sometimes explicitly explained the distance that separates them from Epicurus and his followers. As an example, William of Conches (1997) writes in his *Dialogue on Natural Philosophy* (I, 6, 16):

> When the Epicureans said that the earth consists of atoms, they were correct. But it must be regarded as a fable when they said that those atoms were without beginning and 'flew to and fro separately through the great void', then massed themselves into four great bodies. For nothing can be without beginning and place except God.

Here, we will focus on later debates in the Latin West. For it is only after the translation of the whole Aristotelian corpus, together with some Greek and Arabic commentators, that atomism became a major philosophical topic. The first representative of this tradition is Robert Grosseteste (1175–1253), himself a translator of Aristotle and a major witness of the scientific achievements of his time (McEvoy 2000). Although he continued to be influenced by twelfth-century philosophers, he was marked by the Aristotelian tradition. In order to respond to Aristotle's arguments against atomism in the *Physics*, *On Coming to Be and Passing Away*, and *On the Heavens*, which were completed by Greek and Arabic commentators (Sorabji 1983), Robert Grosseteste introduced a new form of geometric atomism, inspired by the Platonic and Pythagorean traditions, into the larger debate on the nature of the continuum initiated in the Aristotelian tradition against Democritus and Epicurus.

Robert Grosseteste's intuitions were to be followed and developed by some fourteenth-century thinkers at Oxford and Paris, such as Henry of Harclay (c. 1270–1317), Walter Chatton (c. 1290–1343), William Crathorn (fl. c. 1330), Gerard of Odo (c. 1285–1349), Nicolas Bonetus (c. 1280–1343), Nicolas of Autrecourt (1299–1369), Marco Trevisano (d. 1378), and John Wyclif (1320–1384). In the following pages, we will only consider some positive arguments for atomism during this period, leaving aside the powerful critics addressed to them by their contemporaries John Duns Scotus, William of Ockham, Adam Wodeham, William Alnwick, John Buridan, Thomas Bradwardine, or Gregory of Rimini (for an insight into these critics, see Zupko 1993; Cross 1998; and Murdoch 2009).

Continuity and Measure

Robert Grosseteste's atomism is based on two different kinds of considerations. First, according to his Neo-Platonic metaphysics of light, natural bodies were produced at the beginning of the universe by the infinite self-multiplication of a single point of matter and form (1978: 10–11).

The diffusion of the first form was equivalent to that of light when multiplied in all directions from a single source. This is supposed to give to prime matter its corporeity and dimensions. According to this view, the world is constituted of an infinite number of point-like particles of form and matter occupying all possible places, so that there is no vacuum left in this closed universe. This, he affirms, "was the meaning of the theory of those philosophers who held that everything is composed of atoms, and said that bodies are composed of surfaces, and surfaces of lines, and lines of points" (ibid.: 12).

Such a geometric atomism is explicitly attributed to Plato in Aristotle's treatise *On the Heavens*, III.1 (299a2–300a19): ultimate constituents of reality are indivisible points, not primary surfaces, like triangles, for instance. From a metaphysical point of view, the infinite multiplication of light/form into all possible indivisible points of matter is supposed to explain how bodies are made of surfaces, surfaces of lines, and lines of points. In other words, with the division of a continuous body into its ultimate parts, it would return to its elements that were initially multiplied when the world was created.

A second important aspect of Grosseteste's atomism is his theory of measurement (Lewis 2005). Beyond the conventional system of measurement we ordinarily use for spatial or temporal magnitudes, such as miles and kilometers, square feet and square meters, or hours and minutes, continuous magnitudes have absolute sizes according to Grosseteste. These absolute sizes, if they exist, must be measured by a natural and objective system of units. In his *Notes on the Physics* (1963 IV: 92), Grosseteste suggests imagining a possible world in which only one single line exists: how could it be measured? It would be measured either by itself, or by one of its aliquot parts. Again, how could this part be measured? It would be measured either relatively to the whole or relatively to a smaller part, and so forth. In order to stop this infinite regress, one has to suppose that there exists some natural indivisible standard, according to which the world has been created and ordered from the beginning.

According to Grosseteste, only the numbers of indivisibles contained in the aliquot parts guarantee the objectivity of measurement. Be they continuous bodies, portions of space, or spans of time, magnitudes are therefore measured by indivisibles. Although human beings are obliged to measure a continuum with some aliquot part, because of their natural incapacity to cognize real indivisibles, the divine mind, which our author considers to be the "first measurer," knows exactly all these units, their number, how they are ordered, and therefore the absolute size of any entity in the natural world. This is true for spatial magnitudes, which are measured by sizeless points, but also for time, which is measured by indivisible instants.

At the beginning of the fourteenth century, Henry of Harclay rephrased this as follows:

> It is clear that in order to obtain the primary essential element of measure one must have recourse to what is ultimate and indivisible in numbers, namely to a unit. Thus, one quantity measures another only by means of being a unit, since for one quantity to measure another is for it to be related to it as a unit is related to some number . . . For one does not know how long a line may be from the fact that it contains four of its fourth or five of its fifths (for this we know of any quantity whose measure, however we do not know). Rather, it is necessary to know the quantity of that <fourth or fifth> part, which can be accomplished only by appeal to some simple and indivisible element of the <part>. Consequently, a perfect measure of a continuous quantity exists only by virtue of an indivisible of discrete quantity, namely by virtue of a unit, and similarly by means of an indivisible of continuous quantity, namely, <by means> of a point. Moreover, no quantity can be perfectly measured unless we know how many indivisible points it contains. Yet, since these are infinite in number, they cannot be known by a creature, but only by God, who has disposed *all things in number, weight, and measure* (*Wisdom*, 11).
>
> *(2008: 1027)*

One important consequence of Grosseteste and Harclay's theories is that the difference in size between two continua can be analyzed in terms of the numerical ratio of the infinite numbers of indivisibles they contain. If a segment of a line is twice as long as another one, it contains twice as many indivisibles, even if the two are infinite in number. In other words, one infinite number of indivisibles can be twice as big as another one. As far as we know, Grosseteste is the first medieval author who conceptualized the existence of unequal infinites (Lewis 2012).

To reinforce their position about unequal infinites, Grosseteste and Harclay have recourse to numbers. Let us imagine a world, they suggest, that lasts eternally after its creation. If we now consider the correspondence between the number of years and the number of months, the former is 12 times as great as the latter. Hence, if the world never ceases to exist, the infinite number of years is higher than the number of months, days, hours, etc. Naturally, one could contend that there is a difference between an infinite by addition and infinite divisibility. Moreover, numbers constitute discrete quantities, so that they cannot account for the constitution of the continuum. But Grosseteste and Harclay assume that points in a continuum are like numbers in a discrete quantity. And if some supernatural power—at least God—can add these units with one another, then it is also possible to divide the continuum into these units. Accordingly, if one does not want to abandon the objectivity of measurement, the existence of real indivisibles and the possibility of comparing unequal infinites with some numerical ratio must be admitted without qualification.

Apart from Aristotle's genuine works, the main source of Robert Grosseteste's theory of indivisibles is the Pseudo-Aristotelian treatise *On Indivisible Lines*, which Grosseteste translated himself from Greek into Latin. As we shall see, this neglected text is of particular importance for the history of medieval atomism.

The Pseudo-Aristotelian *On Indivisible Lines*

The treatise begins with a series of five arguments for the existence of atomic parts in a line, the first of which being precisely directed at showing that the defenders of the infinite divisibility of a continuum cannot give a rational explanation of the differences in size among continuous magnitudes (968a2–9). In the same way as Lucretius in his *De Rerum Natura* (1947: I.615–626), the author uses a version of Zeno's paradox of large and small. According to this paradox, the divisibilists should assert that the small and the large, the few and the many, have exactly the same infinite number of parts. Consequently, they cannot account for the differences of size in continuous bodies or geometric figures. The argument concludes that only the large and the many are infinitely divisible, whereas the small and the few are only finitely divisible. But if it were the case, then there should be indivisible units in a finite line, i.e. atomic lines.

The author replies to this argument by affirming that the small is also infinitely divisible. Let us take a small segment S in a line L; if S is still a line, it is continuous and therefore infinitely divisible, at least into points, as the text clarifies (not in a genuine Aristotelian fashion). Therefore, S can be smaller than L and both S and L are infinitely divisible. This solution suggests two things: (1) that the ultimate unit in a line, if there is one, is the point and not an atomic line; (2) that it is possible to maintain the commensurability of two magnitudes even if they are infinitely divisible. This is probably the premise of Grosseteste's view on the measurement and composition of the continuum.

The text, however, also affirms that one should not deal with these questions in the light of numbers, and Grosseteste and Harclay cannot agree with the Pseudo-Aristotelian on this point. On the model of Boethius' *On Arithmetic*, they are prone, indeed, to assert that the science of arithmetic is the only way to understand how the continuum can be at the same time infinitely divisible and composed of indivisibles.

This might have been suggested to Grosseteste by the fifth argument in favor of atomism presented in *On Indivisible Lines* (968b4–21) and by the author's reply (969b3–26). The argument runs as follows: for every two lines that are commensurable with one another, there is a unique and common measure, which is an indivisible line. The Pseudo-Aristotelian argues against this view in two ways: (1) there is no necessity in having a single kind of unit for the measurement of a magnitude; (2) some magnitudes are not commensurable, as it appears in geometry (the ratio of the diagonal to the sides of a square is irrational, for instance). This is, of course, a real challenge for an atomist. But, since Grosseteste admits the divisibility of the continuum into an infinite number of points and the possibility of unequal infinites, he can answer that if the ratio of numbers can be either rational or irrational (a whole number or a fraction), so it can be for magnitudes such as lines, surfaces, and bodies constituted by an infinite number of point-units.

It appears then that Grosseteste and Harclay believed, unlike Ancient atomists, that the natural world is a continuum and a plenum. Like Aristotle, they considered the continuum to be infinitely divisible, but it is infinitely divisible into indivisibles, not into divisible parts. The principal reason for that is the need of a natural and intrinsic measurement for all kinds of magnitudes. A first limit to the theory is the difficulty to understand how numbers, i.e. discrete quantities, can be applied to the continuum. A second is the concept of infinity involved in the discussion, which is not clearly defined. Indeed, Grosseteste and Harclay tend to affirm that to each infinite set of indivisibles corresponds an exact number, only known to God, so that what is infinite for us is in reality finite. Later atomists will be rightly puzzled by this weak and relative concept of infinity. This will lead many of them to subscribe to a form of "finitism," according to which the continuum is only finitely divisible.

The Road to Finitism: Zeno's Metrical Paradox of Extension Revisited

Zeno of Elea is mostly remembered for the four paradoxes of motion discussed in Aristotle's *Physics*, which all deal with the notions of continuity and infinite divisibility (see Sorabji 1983: 321–335). Defenders of atomism also used another Zenonian paradox usually called "the metrical paradox of extension" (Grünbaum 1970) or "the paradox of measure" (Skyrms 1983). Epicurus used it in his *Letter to Herodotus*, but its most complete presentation is found in Simplicius' commentary on the *Physics*. As they ignored these two texts, medieval thinkers might have reconstructed the argument from Aristotle's remarks in *On Coming to Be and Passing Away* (I.2, 316a14–30). Here follows a possible wording of the paradox:

1 Every magnitude M is divisible into n parts.
2 M is equal to the sum of the n parts ($\sum x_1 + x_2 + \ldots + x_n = M$).
3a Either the parts of a continuum have a zero magnitude
3b or a positive magnitude.
4 If (3a), then M should also have a zero magnitude, because from (2), it follows that M is the sum of n zero magnitudes, which is equal to zero ($\sum 0 + 0 + \ldots + 0 = 0$).
5 If (3b), then either
5a M is finitely divisible, and (2) still holds true, or
5b M is infinitely divisible, and (2) no longer applies—at least if M is a finite quantity because:
6 The infinite sum of n positive magnitudes makes an infinite magnitude.

(1)–(3) are explicitly accepted by Grosseteste and Harclay. But if the indivisible units used—by God—to measure a continuous magnitude were sizeless points—be they finite or infinite in number—they could not account for its quantity. However, if one concedes that indivisibles have a minimal magnitude, then, from (2), (5b), and (6), the sum of these indivisibles should correspond to an infinite magnitude, not to the finite sensible object we experience in everyday life. As a consequence of this paradox, both Grosseteste's and Harclay's atomism seem to fail, together with the Aristotelian theory of the continuum.

From a contemporary point of view, the resolution of this paradox would consist in denying the validity of the principle of additivity (2) and more specifically its application to an infinite series of items in (6). Anyhow, no one denied the validity of (2) in antiquity and it is likely that the same is true for a large majority of medieval authors. Another obvious option would consist in admitting that the continuum is made of a finite number of parts with a minimal magnitude. One possibility for defending such a theory is denying that a finite magnitude can be actually divided into and composed of an infinite number of parts. Indeed, following Aristotle's suggestion in *Physics* I.4 (187b13–188a5) and *On Coming to Be and Passing Away* (I.2, 316b33–317a1), one might conceive the existence of minimal parts beyond which no physical division can be made, even though these minima continue to be infinitely divisible, at least potentially, i.e. conceptually and mathematically speaking. As is well known, Epicurus and Lucretius used this distinction in their response to Aristotle's arguments against atomism (Furley 1967: 111–129). Such strategy can be followed along different lines. Physical atoms might be still divisible potentially into divisible parts (Aristotle) or only finitely divisible into indivisible parts (Epicurus). Again, the Epicurean option can be understood in two ways: either the minimal parts of the physical atom have a positive (although indivisible) magnitude or they are sizeless points.

The problem for the Aristotelian option is that the paradox can be repeated for potential parts. For even if the division never occurs, potential parts are conceivably divisible into divisible parts, i.e. with a positive magnitude. With regard to the Epicurean solution, the paradox seems to be blocked by the fact that the number of atoms and minimal parts is finite. But an Aristotelian could still object that if the minimal parts of an atom have some positive magnitude, they are still infinitely divisible, at least potentially. Another option is that the minimal parts of a physical atom are sizeless points. But then, how could one hope to escape from step (4) of the paradox?

A majority of fourteenth-century atomists after Henry of Harclay tended to adopt the view that the continuum is composed of a finite number of minimal physical parts (*minima naturalia*), which are composed of a finite number of mathematical indivisibles. Walter Chatton, Gerard of Odo, William Crathorn, and John Wyclif precisely used the metrical paradox of extension in order to deny the truth of (5b). According to them, if M is a finite magnitude, it is equal to the sum of its *n* parts (2), so that *n* is necessarily a finite number (Robert 2010). So, when Grosseteste and Harclay said that the precise number of indivisibles in a continuum is only cognized by an infinite divine mind, they meant this number to be finite. What is usually left unclear is the nature of the parts of the continuum. Does the division presupposes the parts or creates them?

Division Everywhere: Potential and Actual Parts

At the beginning of *On Coming to Be and Passing Away* (I.2, 316a10–317a12), Aristotle presents one of the arguments Democritus used in favor of his atomism. Suppose, he says, that a body or a magnitude is divisible everywhere and that its division is a real possibility. What is left after the division? Divided parts are either magnitudes, or points, or nothing. They are not magnitudes since they are still divisible. They cannot be points, since the addition of sizeless points cannot give rise to a quantity. Indeed, points cannot touch each other since they have no parts. Finally, they are not nothings, because a figure or a body cannot come out of nothing. Democritus' conclusion was that there must exist atomic bodies and magnitudes, whereas Aristotle contends that bodies and magnitudes are divisible everywhere potentially, though not actually.

In order to respond to this new paradox, Walter Chatton endeavored to define more accurately the ontological status of the parts of a continuum (Murdoch and Synan 1966; Robert 2010).

Potential parts, he asserts, might exist after the division, although they do not exist separately from the whole before division. Actual parts, however, exist on their own, independently of any bigger whole. Hence, according to Chatton, it is a contradiction *in adiecto* to affirm that a continuous body or magnitude is composed of actual entities. If it were the case, the result would be a mere aggregate of independent atomic parts, as in Democritus' theory. In the same way, he continues, one has to criticize Plato's geometric atomism if it is understood literally. Surfaces, lines, and points are not actual parts of the continuum. Rather, one should say that a continuum is composed of potential entities, with no independent being before division. In other words, things are continuous if they really make one (continuity is a property similar to homogeneity).

This is not to say that points do not exist in a line, a surface, or a body. Indeed, Chatton was vigorously opposed to Ockham's "non-entitism," according to which indivisibles do not exist at all (see Wood's introduction in Adam Wodeham 1988). For example, if one imagines a perfect sphere touching a perfect plane on one single point and then rolling on each and every point in that plane. These points of contact really exist, but are only potentially present in that plane and that sphere, not as separate entities. Now, could they exist separately?

First of all, like Aristotle and Epicurus, Chatton accepts the existence of limits in the physical division of a natural body. Division everywhere is not naturally possible, even though division is still possible, potentially, beyond these limits. Nevertheless, Chatton asserts that "potential" means that it is not contradictory with separate existence. In order to establish this point, Chatton adds some new arguments based on God's absolute power. God does not only cognize the precise number of indivisibles, as in Grosseteste's and Harclay's theories, but he can also, without contradiction, remove repeatedly the end-point from a line or add a new one, and this makes a different line, with a different quantity. Grosseteste and Harclay already accepted such symmetry between addition and division. But Chatton goes one step further: God can divide the continuum up to its ultimate parts, so that points would exist as separate entities. By this thought experiment, division everywhere becomes a real possibility, even though it never occurs naturally. Anyhow, with the paradoxes previously exposed, if a finite continuum were divided everywhere, the number of indivisibles would be finite. Therefore, it follows that *minima naturalia* are still divisible into a finite number of points.

Some years later, Gerard of Odo and William Crathorn criticized Chatton for having defined points as mere potential parts (de Boer 2009; Robert 2009). According to them, it implies a too restrictive definition of the continuum, since it does not allow two actual entities to form a new continuous entity. Indivisibles can well be actual parts of the continuum. First, their mereology assumes that a whole W is nothing but the sum of its parts. Second, they claim that if the parts of a continuous whole W are responsible for its existence, they must be real and actual parts, not only potential parts (the possibility of division implies that parts already have actual existence before division). Then, if W were infinitely divisible, it would result that the sum of its parts is an actual infinite. Consequently, W would have contradictory properties (it would be actually finite and actually infinite). Moreover, this would force the divisibilist to accept the existence of an actual infinite (a notion which is not accepted by Aristotle himself). Therefore, if the division of a continuum were to be achieved by some power—at least by God—the number of parts would be finite.

This conception of mereology allows them (and later John Wyclif) to form a new argument based on the metrical paradox of extension. If a whole W is nothing but the sum of its parts, the same is true for its place. The place occupied by W is nothing but the sum of the places occupied by its parts. Now, if you divide a table in two pieces, each piece will occupy half of the place of the table, and this is true for every further division. If W were infinitely divisible, from the division would result an infinite number of places, and this infinite sum of places, with a positive extension, should correspond to an infinite place, not to the finite place occupied by the table.

This position is far from being clear. Both Gerard of Odo and William Crathorn continue to talk about atomic parts as indivisible points, as if they defended a version of Plato's geometric atomism. But their arguments seem to imply a much more physical conception of the atoms: they are actual entities, occupying a particular position in space, and their number is limited. Crathorn even mentions the existence of "points of gold" and "points of lead," as if indivisibles had properties like *minima naturalia*. Anyhow, to be consistent, their solution to the metrical paradox of extension should attribute to indivisibles a minimal magnitude. Unfortunately, they never made clear whether they accept a distinction between physical atoms and mathematical indivisibles.

This adhesion to finitism has some important consequences on their conception of mathematics. Indeed, if the number of indivisibles is finite, how could one answer to the argument of the incommensurability of the diagonal and the side of a square? Avicenna and Al-Ghazali formulated the argument as follows: if one draws a parallel line from each point in one side of a square, each of them will cross the diagonal in exactly the same number of points, so that the diagonal and the side should be composed of exactly the same number of points. This argument, popularized by John Duns Scotus, threatens all kinds of indivisibilism, not only finitism. But all the finitists (Chatton, Odo, Crathorn, and Wyclif) answer this by saying that mathematical arguments only hold for abstract objects, not for physical lines, surfaces, and bodies. The real lines drawn on a sheet of paper are "tortuous" and cross the diagonal in several points, with angles. The diagonal does not have the same size as the side, and they are probably incommensurable (if the number of points is a prime number, for instance).

Later in the 1340s, Nicolas of Autrecourt and Nicolas Bonetus defended more entrenched positions. In his *Small Treatise on Quantity*, Nicolas Bonetus decided to support another theory, that of Democritus. As Bonetus defines it, Democritus' theory is partly in agreement with Aristotle, because the continuum is composed of parts that are themselves quantitative, and partly with Plato, since it is nonetheless ultimately composed of indivisibles. Democritus' atomism also differs from both of them, because atoms are not like mathematical points; they are indivisible bodies. Bonetus clearly states that Democritus' atoms are equivalent to the natural minima allowed by Aristotle's physics. They correspond to the limit of the physical division of a natural body, even though some division is still conceivable. Atoms are physically indivisible, but they have potential parts, which are also indivisible. Properly speaking, these mathematical indivisibles do not enter into the physical constitution of the thing. They are nothing but the terms, the end-points of a segment of a line. Thus, points are real entities, but they have no other existence than of a limit or extremity of a line, a surface, or a body. Bonetus concludes that division everywhere, if possible, would give rise to a limited number of atomic magnitudes, and therefore to a limited number of end-points.

At the same time, in his treatise called *Exigit Ordo*, Autrecourt made use of almost all the arguments for finitism mentioned earlier (Grellard 2004). Like Odo and Crathorn, he affirms that parts of the continuum are actual parts, because the concept of "division" is relational and it implies the pre-existence of the relata. Like Henry of Harclay and his followers, he defines the parts of a continuum by their position (*situalitas*) in space. As he puts it, compared with the whole continuum, points are not quantified, but in so far as they have a definite position and a proper mode of being, they are in some way quantified. So it seems clear that there are only a finite number of these atoms. Nevertheless, at the end of this discussion on indivisibles, Autrecourt asserts that if God divided the continuum eternally, he would probably arrive at an infinite number of indivisibles. This is probable, Autrecourt says, because of the apparent validity of mathematical arguments based on incommensurability. Autrecourt seems to assume this contradiction in the four conclusions he gives at the end of this part of the *Exigit Ordo*: (1) the continuum is not composed of parts which can be further divided; (2) a continuum is not composed of a finite number of indivisibles; (3) for every sensible or imaginable magnitude, there is a smaller one; (4) there is a magnitude such that a smaller one does not exist. It has been argued (Pyle 1995: 204–209) that

(3) only means that the senses and imagination cannot arrive at a real indivisible, whereas it really exists and is not cognizable by our finite mind. Conclusion (4) simply says that (3) is not true for every magnitude, because there are indivisibles that are the ultimate constituents of reality (1). The problem is therefore the acceptance of (3). Andrew Pyle and Christophe Grellard argue that Autrecourt defends a doctrine of the infinitesimal: the ultimate constituents of reality are indivisible, they have an extension > 0, and they are infinite in number (even though the material bodies are only finitely divisible by a natural power).

Concluding Remarks

The first aim of medieval atomists was not to deliver a mechanical explanation of the natural world based on the motion of atoms in an infinite void space. Rather, they were looking for an acceptable description of the natural order and its creation, which would also give a rational account of the finitude of the Aristotelian cosmos as well as an objective system of measurement for magnitudes and material bodies. Of course, their acceptance of atomism had some consequences on their conception of physics. As an example, local motion can be described in terms of indivisible spaces traversed in a number of instants of time. Hence, atoms are in motion, but not without a body and not in a void space (Robert 2012). They also applied atomism to other types of motion, such as rarefaction and condensation, and more generally to qualitative variations, such as degrees of heat, for instance. Thus, it would be misleading to consider medieval atomism as a purely mathematical or geometrical theory, since it is also designed to give an account of physical phenomena.

What is striking in this theory is that even though they used discrete quantity in order to describe the continuum, medieval atomists were definitely continuists. Bodies, geometric figures, space, time, and motion are continuous. This seems to be a vain wish. For, as Aristotle puts it, two things form a continuum when their extremities are together, but indivisibles have no extremities, so that they cannot make a continuum. They cannot be in contact with each other, since, as Aristotle says in *Physics* VI, atoms would touch either whole to whole, or parts to whole, or parts to parts. As they do not have parts, they necessarily touch whole to whole, which means that they are superposed and do not create an increase in size (Murdoch 1964). From Henry of Harclay to John Wyclif, all the atomists employed the same strategy to answer this argument. Aristotle's claim is only true if we consider indivisibles or points that are in the same place. If one can distinguish their position (*situs*) in the continuum, it is not contradictory that points might be contiguous, without any point between them. Let us take, for instance, a point defined by the meeting of two perpendicular lines. If one of the two lines is moved from the left to the right, the point will be at different positions in the second line. This is true for both the divisibilists and the indivisibilists who accept the reality of points. It is therefore conceivable from a mathematical point of view that points have a definite position in space, so that they can be next to each other without any gap, and not superposed. In order to understand this claim, one has to move from sense experience to imagination, from physics to metaphysics. As we have seen, medieval atomists regularly ask the reader to imagine the physical world by assuming the view of God, whose power is only limited by the principle of non-contradiction.

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