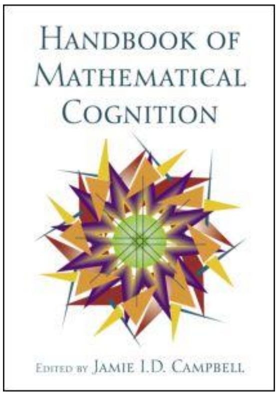

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Spatial Representation of Numbers

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Spatial Representation of Numbers

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INTRODUCTION

Intuitively, we think of number processing as an abstract and nonspatial cognitive activity. Apart from those skills necessary for mental symbol manipulation, no spatial processing seems to be involved in numerical operations. A closer inspection, however, shows that spatial and number processing are intimately connected. A link between mathematical abilities and spatial skills has been anecdotally reported in the past. Great mathematicians like Einstein explicitly emphasized the role of visuo-spatial imagery for the development of their mathematical ideas (cf. Hadamard, 1945/1996). About 15% of normal adults report visuo-spatial representations of numbers (Galton, 1880a,b; Seron et al., 1992). This suggests that the integration of number representations into visuo-spatial coordinates is not a rare phenomenon. The reported spatial layouts were predominantly oriented from left to right, were mostly automatically activated, were stable in time and had emerged in childhood.

More systematic studies have supported these anecdotal reports by demonstrating a tight correlation between mathematical and visuo-spatial skill. In the clinical field, learning disorders establish a similar association between visuo-spatial and mathematical disabilities (e.g., Rourke & Conway, 1997). Evidence from brain imaging provides further support for a link between numbers and space. Tasks that require either number processing or spatial transformations tend to activate structures within the parietal lobes (Milner & Goodale, 1995; Dehaene et al., 2003). Using transcranial magnetic stimulation in healthy participants, Göbel et al. (2001) showed that stimulation of the left and right parietal cortices leads to decreased performance in both visuo-spatial search and number comparison tasks. This suggests that the processing of numerical magnitudes and of visuo-spatial information are functionally connected. Patient studies further confirm the close link between visuo-spatial processing and basic number processing. A particular example is Gerstmann syndrome, which is characterized by the co-occurrence of left–right confusion, finger agnosia, and dyscalculia (e.g., Dehaene & Cohen, 1997).

Thus, there appears to be a convincing case for a link between numbers and space. None of the above reports does, however, force the conclusion that truly numerical representations or processes are associated with spatial representations. The observed correlation could instead reflect the involvement of shared peripheral support structures. For example, visuo-spatial working memory is engaged in symbol manipulation during mental arithmetic (Lee & Kang, 2002). In this chapter we will report evidence that semantic representations of number magnitude are indeed spatially defined and can be conceptualized as positions on an oriented *mental number line*. The idea of a linear analogue representation of numbers in the mind has been proposed (e.g., Moyer & Landauer, 1967; Restle, 1970) to account for some basic performance patterns in numerical cognition. More recently, this useful metaphor has been augmented by postulating that the hypothetical mental number line also has a spatial orientation. We will also show that this spatial cognitive representation of numbers should not be considered as fixed and unchangeable, by demonstrating that the characteristics of spatial number coding are largely determined by numerical and spatial parameters specific to the task at hand. Moreover, the spatial coding of numbers is not under strategic control but rather occurs automatically.

MENTAL REPRESENTATION OF NUMBER MAGNITUDE IS SPATIALLY CODED: THE SNARC EFFECT

Mental chronometry involves the timing of behavioral responses in simple cognitive tasks. Using this approach, Dehaene et al. (1990) asked their participants to indicate with a left or right key press whether a visually presented probe number was smaller or larger than a previously announced reference number. For example, randomly drawn probe numbers from 1 to 99 (but excluding 55) would be compared against the fixed reference number 55. The decision speed in this *number comparison task* with fixed reference was recorded and analyzed as a function of the probe number's magnitude and the response side. Participants who had to press the left key to indicate a "smaller" response and the right key to indicate a "larger" response were faster than those who had to respond left for "larger" and right for "smaller" probe numbers. This response side effect suggested that number magnitude is represented on a left-to-right oriented mental number line, with small numbers on the left side and larger numbers further on the right side. In a seminal paper, Dehaene et al. (1993) further explored this observation.

Dehaene et al. (1993) asked their participants to decide, by using a left or right key, whether a single number was odd or even. In the basic version of this *parity task*, the digits from 0 to 9 appeared repeatedly in a random order in central vision. Different response rules (odd number—left button, even number—right button; or even number—left button, odd number—right button) were tested in counterbalanced blocks. In this way, each participant's response speed as a function of number magnitude could be evaluated. Statistical analysis of the reaction times (RT) revealed that small numbers were responded to faster with the left key, whereas large numbers consistently showed a right key advantage. Dehaene et al. (1993) named this association of numbers with spatial left–right response coordinates the *SNARC effect* for Spatial–Numerical Association of Response Codes.

The SNARC effect is of key importance for the current issue of spatial coding of numbers. It unequivocally demonstrates that numerical magnitude information is spatially coded in most people. The SNARC effect, as an index of the spatial attributes of number representations, has led to several studies into the nature of the mental number line. Below, we will review these studies and their implications. But first we discuss the measurement of the SNARC effect.

Figure 3.1a shows that the SNARC effect can be expressed as a statistical interaction between number magnitude and response side. But because the SNARC effect reflects an association between the position of a number on the mental number line and the position of a response key, we can assess this spatial association more effectively with a statistical regression

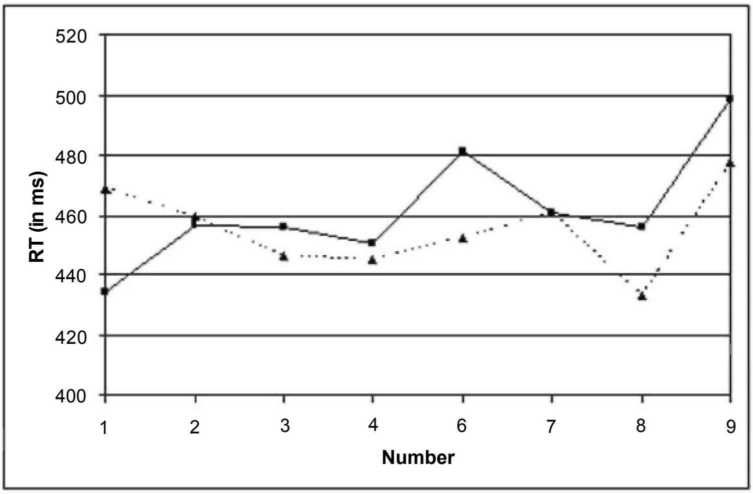

Figure 3.1a. Typical SNARC effect presented as an interaction between number magnitude and side of response (dotted line: right-hand responses; full line: left-hand responses).

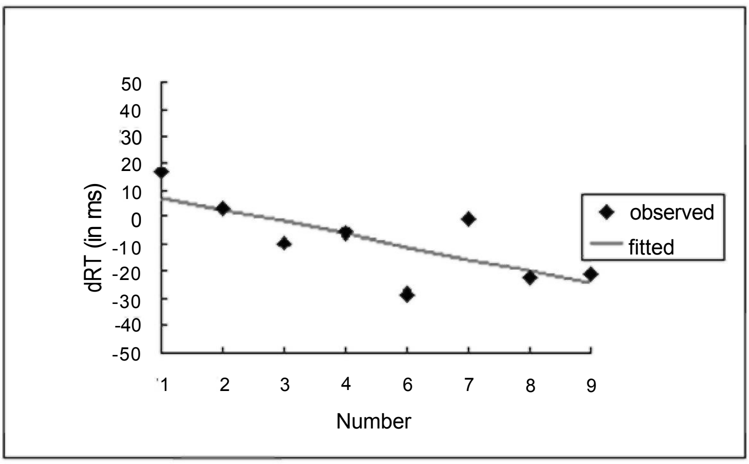

Figure 3.1b. The same SNARC effect presented as a linear regression line with negative slope that is fitted through the difference scores dRT for each stimulus digit.

analysis (Fias et al., 1996). Specifically, the difference in RTs (dRT) for right minus left key responses will be positive for small numbers and negative for larger numbers (see Figure 3.1b). The most straightforward way to capture this negative correlation between numbers and space statistically is to regress dRT on number magnitude for each participant and to then test the slope coefficients against zero (Lorch & Myers, 1990).[1]

There are several advantages related to this regression-based analysis of the SNARC effect. First, the presence of a SNARC effect is judged by a main effect (does the averaged slope coefficient obtained from individual regression equations differ from zero?) rather than by the presence of an interaction between magnitude and side of response. Second, number magnitude is considered as a continuous variable. Third, the regression analysis allows a straightforward

[1]A participant's hand dominance has no effect on the overall pattern but can affect the intercept of the regression line.

quantification of the size of the effect (how steep is the slope?) rather than a mere qualitative judgment about the presence or absence of an interaction. Fourth, the effect of additional variables can easily be partialed out through statistical techniques. Fifth, the method evaluates the linear relation between number magnitude and dRT for each participant, reducing the chance of misestimating the SNARC effect due to group averaging. This also allows researchers to explore the influence of individual-specific variables such as gender or handedness on the association between numbers and space. Finally, the method is more flexible than other approaches because it does not require an orthogonal combination of the experimental factors. This is of interest when investigating other tasks than parity judgments that do not rely on the sequential alternation between number magnitudes and response codes.

SPATIAL NUMERICAL CODING IS DYNAMIC: NUMERICAL AND SPATIAL DETERMINANTS OF THE SNARC EFFECT

To obtain a detailed understanding of the association between numbers and space and, by extension, the properties of the mental number line, it is important to know which numerical and spatial variables determine the SNARC effect. We therefore review the recent literature from this perspective.

Numerical Determinants of the SNARC Effect

Several studies have shown that the spatial coding of numbers depends on the task context. The SNARC effect has most frequently been studied in parity tasks with Arabic digits from 0 to 9. An important observation emerged from manipulating the range of stimulus digits: when the *range* of digits was either 0–5 or 4–9 in separate conditions (Dehaene et al., 1993, Experiment 3; see also Fias et al., 1996), the digits 4 and 5 were associated with right responses when they were the largest digits but with left responses when they were the smallest digits to be judged. This shows that the spatial association for a given number is between its *relative* magnitude and space.

An obvious extension is to ask whether the spatial associations also hold for multi-digit numbers. Dehaene et al. (1993) used digits from 0 to 19 and found that the SNARC effect did not clearly extend toward the two-digit numbers. This suggests that the mental number line might be restricted to the representation of single-digit numbers. However, before accepting this conclusion, it is important to realize that the parity status of a two-digit number is determined by the rightmost digit. Parity judgment RTs in Dehaene et al.'s (1993) experiment were indeed largely predictable from the rightmost digit, indicating that the participants had adopted this selective attentional strategy. More informative with regard to the issue of multi-digit spatial representations is the earlier magnitude comparison study of Dehaene et al. (1990), in which probe numbers smaller than the reference were responded to faster with the left hand than with the right hand and vice versa for larger numbers, indicating spatial coding of two-digit numbers. Using another variant of the SNARC effect, Brysbaert (1995) also found a SNARC effect for two-digit numbers, which were processed more quickly when the smaller number was to the left of the larger number compared to a display with the larger number on the left.

Together, these results indicate that number meanings conveyed by single-digit as well as by two-digit numbers are spatially coded. It remains, however, unclear whether the mental number line is a single, analogue continuum onto which various number intervals can be projected as required or whether there are separate mental representations for single- and multi-digit numbers. At this point, it is also unresolved whether two-digit numbers are processed holistically or compositionally. Initially, holistic processing was assumed (Brysbaert, 1995; Dehaene et al., 1990; Reynvoet & Brysbaert, 1999), but recently evidence is accumulating for a separate representation of decade and unit magnitudes during the processing of two-digit numbers (Fias et al., 2003; Nuerk et al., 2001). How both separate and holistic effects should be incorporated into a single processing model is not clear at present. At the very least, effects of stimulus manipulations in number tasks point to a considerable flexibility in accessing the cognitive representation of numbers.

Related to the issue of two-digit processing is the possible extension of the mental number line to *negative numbers*. In western cultures, negative numbers are frequently displayed to the left of positive numbers on the abscissas of statistical graphs. As a consequence of this, we might develop an association of negative numbers with left space. On the other hand, one could argue that negative numbers can be represented more economically on the basis of positive entries alone. The empirical evidence on this issue to date is inconsistent. Fischer (2003a) asked participants to select the numerically larger of a pair of digits ranging from −9 to 9 and measured their decision times in this magnitude comparison with variable reference. Negative numbers were associated with left responses and positive numbers with right responses, supporting the learned association hypothesis. However, pairs of negative digits incurred additional processing costs when compared to mixed or positive pairs, thus suggesting that an additional processing step might have been involved. Moreover, Nuerk et al. (2004) found no reliable spatial association with negative numbers in a parity task. Finally, Fischer and Rottmann (2004) found that large negative magnitudes were associated with right and not left space when a parity task was used but that negative numbers became associated with left space when digits from −9 to 9 had to be classified relative to zero as the fixed reference value. Thus, the spatial associations of negative numbers may be less automatized compared to those of positive numbers.

We now turn to a discussion of the role of number *format* for the spatial association of numbers. Numerical information can be conveyed in many ways, e.g., with Arabic or Roman symbols; in the form of finger postures, dot patterns, or number words; and using either the visual, auditory, or tactile modality. If the SNARC effect indicates access to the abstract representation of number magnitude then it should be insensitive to these variations (see also chapter 2 on this issue). Several studies have obtained SNARC effects when numbers were presented either as Arabic digits or as written words (e.g., Fias, 2001; Dehaene et al., 1993; Nuerk et al., 2004). The slopes of the SNARC functions had similar magnitudes (although sometimes they tended to be smaller for number words), in agreement with the idea that the spatial association reflects access to an abstract representation of number magnitude. Although we know of no published SNARC studies with other number formats (e.g., Roman or Chinese numerals, dot patterns, counting fingers, auditory or tactile magnitude information), further support for a supramodal number representation comes from priming studies, where in each trial a task-irrelevant prime appears before the task-relevant probe number. The typical finding is that decision speed is fastest when the prime and probe are identical, and RT gradually increases with increasing numerical distance between prime and probe. Importantly, this distance effect is not affected by whether the prime and probe numbers are presented in the same or in different formats (Reynvoet et al., 2002).

Finally, it is worth considering whether spatial associations are exclusively numerical or whether they can occur with *non-numerical stimuli* that are sequentially ordered (e.g., letters of the alphabet, days of the week, months of the year). An initial study (Dehaene et al., 1993, Experiment 4) found no reliable associations between letters and space when participants classified letters from the beginning or end of the alphabet as vowels or consonants (see also Fischer, 2003b). However, a statistically more powerful study (Gevers et al., 2003) found that both letters of the alphabet and months of the year can exhibit a SNARC effect. This raises the question: Which aspect of numerical information is spatially coded? Numbers do not only convey quantity information (three buses), but also ordinal information (the third bus) or even nominative information (Bus line 3). It is possible that these different number meanings are conveyed by different representational systems. Given that both numbers and ordered sequences can elicit a SNARC effect, one could argue that it is the ordinal property and not the quantitative property of numbers that is spatially coded. Alternatively, ordinal and quantitative information may be represented separately but characterized by similar internal properties (chapter 4 discusses the processing of ordinal information). Another possibility is that a shared representation can handle numerical or ordinal information, depending on the task context, because quantitative information hierarchically implies ordinal information. In support of this possibility, Marshuetz et al. (2000) found that brain areas which responded to ordinal attributes of non-numerical stimuli were also engaged during number-processing tasks.

Spatial Determinants of the SNARC Effect

In general, spatial information can be coded with respect to a variety of *reference frames*: either centered on part of an observer's body (egocentric coding) or on some non-bodily object (allocentric coding). To investigate the reference frame(s) involved in the SNARC effect, Dehaene et al. (1993, Experiment 6) asked participants in a parity task to respond with crossed-over hands, the left hand pressing the right key and the right hand pressing the left key. Large numbers were classified faster with the right key/left hand and small numbers were classified faster with the left key/right hand. This shows that the relative position of the response, and not the responding hand, determines the SNARC effect. This conclusion is supported by studies involving unimanual responses. Kim and Zaidel (2003) obtained a SNARC effect when participants responded with two fingers of one hand. Fischer (2003b) obtained a SNARC effect when participants classified digits as odd or even by pointing with one hand to a left or right button.

The SNARC effect can be obtained for *effectors* other than the hand and in tasks other than selecting one of two buttons. For example, the time to initiate eye movements away from centrally presented digits to the left or right side (as a function of parity status) depends on the relation between the digit's magnitude and the direction of the eye movement (Fischer et al., 2004; Schwarz & Keus, 2004). Two further results from these oculomotor studies suggest that the SNARC effect emerges at a processing stage prior to effector selection. First, Fischer et al. (2004) showed that the saccadic amplitude is not influenced by the magnitude of the presented number. Second, Schwarz and Keus (2004) found equally sized SNARC effects when comparing manual and oculomotor versions of the parity task.

Bächtold et al. (1998) demonstrated that not only the spatial coordinate system of the response but also the *internal representation* of the numerical information is important. They instructed participants to think of the digits as either lengths on a ruler or times on an analogue clock face. The same digits were then associated with either left or right space, depending on the ruler or clock face condition. For instance, a small number was preferentially responded to with the left hand in the ruler condition but with the right hand in the clock face condition. A similar conclusion can be drawn from two descriptions of brain-damaged patients with hemi-neglect whose impairment to attentively process left space was reflected in their mental representation of numbers. In the first study, Zorzi et al. (2002) observed a systematic representation-based midpoint shift toward the right in a number interval bisection task. For instance, their patients named 6 as the number in the middle between 3 and 7. Apparently, because they were neglecting the left side of their mental number line, these patients positioned the midpoint of a verbally presented interval towards the right. In the second report, Vuilleumier et al. (2004) studied how a group of patients neglecting the left side of space compared numbers to a fixed reference. The patients were selectively slow in responding to the number just smaller than the reference, indicating difficulties in orienting attention towards the left on their mental number line. This selective difficulty was observed for different references (5 and 7). When asked to imagine whether the presented target number was earlier or later than 6 o'clock, the patients showed the reverse effect, a selective slowing of numbers larger than 6, thereby further confirming the dynamic and representational nature of the association between numbers and space

To conclude, the SNARC effect does not seem to tap into a fixed component of the long-term representation of numbers. Rather, numerical information can be dynamically allocated to different representationally defined reference frames, with the left–right line-like spatial coding being merely a default.

A BROADER PERSPECTIVE: THE SNARC EFFECT IN RELATION TO OTHER SPATIAL COMPATIBILITY EFFECTS

Generally speaking, the SNARC effect is the result of joint activation of the spatial components of the cognitive representation of number meaning (magnitude) and of spatial task requirements. More specifically, both the mental number line and the response requirements of certain number tasks share a left–right code. Its congruent activation seems to cause the effect. This makes the SNARC effect a special instance of a *spatial compatibility* effect. Spatial compatibility refers to the fact that lateralized responses can be emitted faster and, less error prone when the trigger stimulus is lateralized to the same side (Fitts & Seeger, 1953). Various types of spatial compatibility can be distinguished as a function of the involvement of spatial aspects in relevant and irrelevant stimulus attributes and in response components of the task (see Kornblum et al., 1990, for a taxonomy). The SNARC effect seems structurally similar to the established Simon effect (Simon, 1969). To obtain the *Simon effect*, participants are asked to give a left- or right-key response to a nonspatial task-relevant attribute of a stimulus (e.g., its color) which is presented randomly either left or right of fixation. This task-irrelevant spatial information contained in the stimulus position then influences the response: right-key presses are slowed down when stimuli appear on the left compared to the right side, and vice versa for left key presses. In SNARC experiments, stimuli are presented centrally and the task-relevant information (typically parity status or magnitude) is also nonspatial in nature. Nevertheless, a task-irrelevant spatial attribute seems to become activated from the internal number representation and to then either facilitate or interfere with the spatial processing required to respond.

The compatibility effects obtained with internally represented spatial dimensions and externally presented spatial stimulus attributes seem to have a similar origin. For instance, Masaki et al. (2000) showed that the compatibility effect with centrally presented arrows (conveying spatial information symbolically) evoked a pattern of electrophysiological brain potentials that highly resembled the pattern obtained with the traditional Simon paradigm (e.g., De Jong et al., 1994). This interpretation is, however, not supported by a recent study of Mapelli et al. (2003). To look for interactions between the SNARC and the Simon effect, they presented digits to the left or right of fixation for parity classification. Thus, they introduced a numerical version of the Simon task, in which the spatial position of the number stimulus was task irrelevant. If the SNARC effect, like the Simon effect, is indeed originating from a common processing stage, then one would expect a statistical interaction between magnitude and position of the digits (Sternberg, 1969). Mapelli et al. (2004), found no such statistical interaction. On the other hand, researchers recently demonstrated interactions between the SNARC and the Simon effects (e.g., Caessens et al., 2003; Wood et al., 2004), suggesting that, like the Simon effect, the SNARC effect results when selecting a spatial response on the basis of task-relevant information and an automatically induced spatial bias. Moreover, in a recent study, Gevers et al. (2004) demonstrated that the SNARC effect was characterized by the same electrophysiological correlates of response selection as observed by Masaki et al.

However, to consider the SNARC effect as an instance of the Simon effect, it is important to demonstrate that the spatial coding of numerical information occurs automatically. We now turn to evidence supporting such automaticity.

Although the SNARC effect has been primarily investigated with the parity task and to a lesser extent with magnitude comparison, the effect is clearly not specific to these tasks. Participants in the study by Fias et al. (1996), for instance, indicated whether the name corresponding to a visually presented digit contained an/e/-sound or not by pressing a left or right response key. Fias et al. found a robust SNARC effect in this phoneme monitoring task. Huha et al. (1995) also observed a SNARC effect when participants evaluated the appearance of visually presented digits. Fischer (2001) reported that the perception of the midpoint of long strings made from small or large digits was shifted to the left or right, depending on the digit magnitude. Finally, participants respond faster with a left button to 1 than to 100 and faster with a right button to 100 than to 1 (Tlauka, 2002), again illustrating how perceptual tasks induce spontaneous semantic processing that is then reflected in a SNARC effect.

Some of the tasks reviewed above required no explicit number-related information to be performed. However, despite the fact that number magnitude was not needed, the numbers had to be processed to some degree. The SNARC effect, however, has also been obtained in studies in which the visually presented numbers were completely irrelevant. For instance, using digits as a background upon which oriented lines or triangles were superimposed for classification, Fias et al. (2001) found that participants' manual responses were influenced by the spatial–numerical association evoked by the background. This is a strong argument in favor of automatic spatial coding. Also, in Fischer et al.'s (2003b) study of visual–spatial attention allocation, the digits served merely as a fixation point but did nevertheless influence speed of target detection. The fact that the SNARC effect emerges when information about numbers is not required for correct performance, and may even interfere with performing the task, suggests that a high degree of automaticity is involved in the processes that give access to the magnitude representation and its spatial association (cf. chapter 4).

To sum up, the SNARC effect in its pure form expresses an overlap in the cognitive representations of the spatial left–right dimensions from the irrelevant number magnitude and the required response and thus fits the category of Simon-like effects in Kornblum et al.'s (1990) taxonomy of compatibility effects. We believe that it is a theoretically fruitful approach to put the investigation of the spatial coding of numerical information within the theoretical frameworks developed to understand general spatial compatibility effects. This leads to two advantages. First, by understanding the domain-general components of the SNARC effect, the number-specific components can be isolated and therefore better understood. Second, a framework is provided to understand spatial coding of numbers in its different manifestations.

DEVELOPMENTAL AND CULTURAL DETERMINANTS

If we want to understand how the association between numbers and space comes about, it makes sense to look at the way children deal with magnitude information.

Developmental studies have shown that very young infants can discriminate numerosities and continuous magnitudes and even perform simple additions and subtractions (Wynn, 1998; see also chapter 9). Following these findings, a debate arose about the functional origin of this precocious numerical ability. Some authors adhere to the idea that these abilities reflect the operation of a "number sense" (e.g., Dehaene, 1997), whereas others suggest that these abilities are not truly numerical in nature but reflect the operation of early visuo-spatial abilities (Newcombe, 2002).

Further evidence for the involvement of spatial cognition in numerical abilities can be obtained at later stages of a child's development. From the work of Rourke and Conway (1997), it is known that visuo-spatial learning disorders correlate with a delayed or abnormal development of mathematical skills. The same correlation has been observed in genetic disorders like velocardiofacial syndrome (Simon et al., 2003) and Williams syndrome (e.g., Ansari et al., 2003; see also chapter 17). These observations demonstrate a prominent role of visuo-spatial abilities in number processing, but they do not clarify how numerical representations become spatially coded. We must therefore turn to the available evidence from developmental and cross-cultural studies on the SNARC effect. Berch et al. (1999) investigated the onset of the SNARC effect with the parity task. They found that the SNARC effect appeared from third grade. However, given the evidence for well-developed spatial and numerical skills in much younger children (see above), it could be argued that the parity task is not sensitive enough to discover the presence of such associations in younger children; they may be unable to respond consistently in this speeded task. The use of behaviorally simpler tasks such as detection (Fischer et al. 2003b) or bisection (Fischer, 2001) may reveal spatial–numerical associations even in such special populations. Alternatively, it might also be that the number line is spatially coded from an earlier age but that it is not yet automatically activated. Remember that the parity task does not necessarily require magnitude information. Consistent with this idea, Girelli et al. (2000) showed that number magnitude is only activated automatically from third grade onward. In sum, further research is needed to establish the critical developmental period for the SNARC effect.

What determines the left–right orientation of the mental number line? One prominent proposal has been that the effect reflects acquired reading habits (Dehaene et al., 1993). Western participants in number studies typically read from left to right, and this cognitive strategy may transfer from the domain of letter, word, and sentence processing to the processing of digits, numbers, and equations. In support of this view, the association of numbers with space tended to be weaker in a group of Iranian participants who normally read from right to left and who probably would associate small digits with right space and larger digits with left space (see Dehaene et al., 1993, Experiment 7, for details of this trend). A recent series of studies by Zebian (2001) strengthens this conclusion. She found that monolingual Arabic speakers in Beirut process two numbers more easily when the larger number is placed to the left of the smaller number, compared to a display with the larger number on the right. This effect decreased for a group of bilingual Arabic–English speakers (see also Maass & Russo, 2003).

Of course, these studies do not demonstrate directly that writing direction itself is the crucial determinant of the orientation of the number line. With the currently available data, any variable that is correlated with it can have a decisive impact. For instance, one might suspect that the association of numbers with spatial positions is a reflection of early training with number lines in school. Poster boards with printed left-to-right-oriented number lines have been used to teach generations of school children the principles of addition and subtraction (Fueyo & Buschel, 1998). Or it could be an expression of culture-specific general exploration strategies (Dehaene et al., 1993). It may also be worthwhile considering finger-counting habits as a reason for the emergence of associations between numbers and space. Several arguments can be made in support of this hypothesis. First and foremost, finger counting is a universal means of learning to deal with numbers (see Butterworth, 1999, chapter 5). Specifically, it could then be argued that the majority of children in Western countries prefer to enumerate objects on the fingers of their left hand and that this brings about the association of small numbers with left space and larger numbers with right space. Conant (1896/1960, p. 437f) reported that from 206 U.S. school children almost all began to count with their left hand. Clearly more up-to-date and cross-cultural data are needed to evaluate this possibility further.

Having discussed these possible candidates for the acquisition of associations between numbers and space, we wish to briefly draw the reader's attention to one further proposal. In an impressive analysis of mental arithmetic from the viewpoint of embodied cognition, Lakoff and Nunez (2000) show how numerical abilities can emerge from ordinary behavior and daily experiences in a physical world. These become cognitively represented in schemas and are then transferred from their source domain to the target domain of arithmetic through the use of metaphor. To illustrate, consider how basic facts about any object collection (its size and how it is modified by removing and adding elements) can be mapped onto statements about numbers. This has also been illustrated by Cooper (1984, p. 158):

> "Consider number development as learning about the space of number. In this space, one must learn where things are and how to get from one place to another. For purposes of the analogy the locations are specific numerosities and the actions to get from one place to another are additions and subtractions. How do you get from two to five? You must start in a particular direction (increasing numerosity) and go past certain landmarks (three and four) until you arrive at five (having gone a certain distance). Points in this space capture the cardinal characteristics of number: direction and landmarks, their ordinal properties . . . It is through experiences of moving in this space that children learn its ordinal structure, which is the primary content of early number development."

Lakoff and Nunez (2000) elaborate how such concrete experiences yield all the laws of arithmetic, such as preservation of equality, symmetry, transitivity, and inverse operations. Their theory, primarily based on arguments from structural and logical analysis, may become a promising avenue for further theory development if put in an empirically testable theoretical framework. We refer the reader to chapter 7 by Nunez and Lakoff for more details.

In sum, there is now good evidence that the direction of the number line is culturally determined, although it remains unclear what the crucial variables are. Further developmental research in a cross-cultural perspective can increase our understanding of the developmental trajectory and the cultural determination of how space is integrated in our internal mental representations of numbers.

CONCLUSIONS

We hope that this chapter has convinced the reader that the meaning of numbers is indeed spatially coded and that the mental number line is a useful metaphor to capture this surprising fact. However, this metaphor should not be taken literally, as there is no sign of a topographic organization of number-selective neurons in the brain (Nieder et al., 2003; Verguts & Fias, 2004). Rather, spatial associations are attached to numbers as part of our strategic use of knowledge and skills, and, as a result, these associations are highly task-dependent. Further evidence of this flexibility of spatial associations challenges the appropriateness of the number line metaphor. Examples include the existence of vertical as well as horizontal spatial associations (Schwarz & Keus, 2003) and the systematic association of odd numbers with left space and even numbers with right space (Nuerk et al., 2003). Future research will have to determine the extent to which the wide range of spatial numerical associations can help us understand the cognitive representation of numbers.

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