Dudley's Handbook of Practical Gear Design and Manufacture

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Gear Tooth Design

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4 Gear Tooth Design

Stephen P. Radzevich

In this chapter the subject of the design of the gear tooth shape is covered. To the causal observer, some gear teeth appear tall and slim while others appear short and fat. Gear specialists talk about things like the “pressure angle”, long and short addendum, root fillet design, and the like. Obviously, there is some well-developed logic in the gear trade with regard to how to choose and specify factors related to the gear tooth shape. This chapter presents the basic data that are needed to exercise good judgment in gear tooth design.

4.1 BASIC REQUIREMENTS OF GEAR TEETH

Gear teeth mesh with each other and thereby transmit non-slip motion from one shaft to another. Those, making and using gear teeth may expect the teeth to conform to some standard design system. If this is case, the gear maker may be able to use some standard cutting tools that are already on hand. If a standard design is used that is already familiar to the gear user, the functional characteristics of the gears, such as relative load-carrying capacity, efficiency, quietness of operation, and the like, can be expected to be similar for new gear drives to those of gear drives already in service.

It should be kept in mind that the gear art has progressed to a point where there is much more versatility than previously. Machine tools are computer-controlled and can be programmed to cover much more variations than were possible in the past. In the gear design office, computers can find what may be believed to be truly optimum design for some important application. Frequently, the design chose does not agree with a design that might have been considered standard in earlier years.

What this means is that a great variety of gear tooth designs are being used. This trend in gear engineering can be a good one—providing the gear designer has an in-depth knowledge of all the things that need to be considered. Serious mistakes, though, can be made when a decision is made by computer data and the computer program failed to consider some critical constraints in the application.

4.1.1 DEFINITION OF GEAR TOOTH ELEMENTS

Many features of gear teeth need to be recognized and specified with appropriate dimensions (either directly or indirectly).

Spur and helical gears are usually made with an involute tooth form. If a section through the gear tooth is taken perpendicular to the axis of the part, the features shown in Figures 4.1 and are revealed. Note the nomenclature used.

Outside diameter (dashed line in Figure 4.1) is the maximum diameter of the gear blank for spur, helical, worms, or worm gears. All tooth elements lie inside this circle. A tolerance on this diameter should always be negative.

Modification diameter (dashed line in Figure 4.1) is the diameter at which any tip modification is to begin. It is a reference dimension and may be given in terms of degrees of roll.

Pitch diameter (dashed line in Figures 4.1 and 4.2) is the theoretical diameter established by dividing the number of teeth in the gear by the diametral pitch of the cutter to be used to produce the gear. This diameter can have no tolerance.
**Limit diameter** (dashed line in Figure 4.1) is the lowest portion of a tooth that can actually come in contact with the teeth of a mating gear. It is a calculated value and is not to be confused with form diameter. It is the boundary between the active profile and the fillet area of the tooth.

**Form diameter** (dashed line in Figure 4.1) is a specified diameter on the gear above which the transverse profile is to be in accordance with drawing specification on profile. It is an inspection dimension and should be placed at a somewhat smaller radius than the limit diameter to allow for shop tolerances.

**Undercut diameter** (dashed line in Figure 4.1) is the diameter at which the trochoid producing undercut in a gear tooth intersects the involute profile.
**Base-circle diameter** (dashed line in Figures 4.1 and 4.2) is the diameter established by multiplying the pitch diameter (see above) by the cosine of the pressure angle of the cutter to be used to cut the gear. It is a basic dimension of the gear. [Note: In low-tooth-count gears, the base-circle diameter is greater compared to its root diameter, and is smaller in gears with regular tooth count (for involute gears with standard tooth profile, gears with 41 tooth and fewer, the base-circle diameter is greater compared to the root diameter, and for gears with 42 and more tooth it is smaller compared to the root diameter)].

Root diameter (dashed line in Figures 4.1 and 4.2) is the diameter of the circle that establishes the root lands of the teeth. All tooth elements should lie outside this circle. The tolerance should be negative.

Figure 4.3 shows a side view of a spur gear tooth to further depict the features and nomenclature of gear teeth.

**Active profile** (shaded area in Figure 4.3,a) is a surface and is that portion of the surface of the gear tooth which at some phase of the meshing cycle contacts the active profile of the mating gear tooth. It extends from the limit diameter (see Figure 4.1) near the root of the tooth to the tip round (see Figure 4.2) at the tip of the tooth and, unless the mating gear is narrower, extends from one side of the gear or edge round (see Figure 4.3,k) at one end of the tooth to the other side of the gear or edge round.

**Top land** (shaded area in Figure 4.3,b) is a surface bounded by the sides of the gear (see Figure 4.3,d) and active profiles; or if the tooth has been given end and tip rounds (see Figure 4.3,h and 4.3,i), the top land is bounded by these curved surfaces. The top land forms the outside diameter of the gear.

**Fillet.** The fillet of a tooth (shaded area in Figure 4.3,c) is a surface that is bounded by the form diameter (see Figure 4.1) and the root land (if present) (see Figure 4.3,e) and by the ends of the teeth. In full-fillet teeth, the fillet of one tooth is considered to extend from the center line of the space to the form diameter.

**Sides of gear.** The sides of gear (shaded area in Figure 4.3,d) are surfaces and are the ends of the teeth in spur and helical gears.

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**FIGURE 4.3** Nomenclature of Gear Tooth Details.
Root land (also known as bottom land). The root land (shaded area in Figure 4.3,e) is a surface bounded by fillets (see Figure 4.3,c) of the adjacent teeth and sides of the gear blank.

Transverse profile (heavy line in Figure 4.3,f) is the shape of the gear tooth as seen in a plane perpendicular to the axis of rotation of the gear.

Axial profile (heavy line in Figure 4.3,g) is the shape of the gear tooth as seen in a plane tangent to the pitch cylinder at the surface of the tooth. In the case of helical gears, it is the shape of a tooth as seen on a pitch cylinder and may be developed to be shown in a plane.

Tip round (shaded area in Figure 4.3,h) is a surface that separates the active profile and the top land. It is sometimes applied to gear teeth to reduce the chance of chipping, particularly in the case of hardened teeth. It may also be added as a very mild and crude form of profile modification.

End round (shaded area in Figure 4.3,i) is a surface that separates the active profile and the top land of the tooth. It is sometimes applied to gear teeth to reduce the chance of chipping, particularly in the case of hardened teeth.

Fillet radius (shaded area in Figure 4.3,j) is the minimum radius that a gear tooth may have.

End round (shaded area in Figure 4.3,k) is the surface that separates the active profiles of the teeth from the sides of the gear. These edges are of importance in the cases of helical gears, spiral bevel gears, and worms, since they become very sharp on the leading edge.

4.1.2 Basic Considerations for Gear Tooth Design

Gear teeth are a series of cam surfaces that contact similar surfaces on a mating gear in an orderly fashion. In order to drive in a given direction and to transmit power or motion smoothly and with a minimum loss of energy, the contacting cam surface on mating gears must have the following properties:

- The height and the lengthwise shape of the active profiles of the teeth (cam surfaces) must be such that, before one pair of teeth goes out of contact during mesh, a second pair will have picked up its share of the load. This is called continuity of action.
- The shape of the contacting surfaces of the teeth (active profiles) must be such that the angular velocity of the driving member of the pair is smoothly imparted to the driven member in the proper ratio. The most widely used shape for active profiles of spur gears and helical gears that meets these requirements is the involute curve. There are many other specialized curves, each with specific advantages in certain applications. This subject is developed further in the section, “conjugate action”.
- The spacing between the successive teeth must be such that a second pair of tooth-contacting surfaces (active profiles) is in the proper position to receive the load before the first leave mesh.

Continuity of action and conjugate action are achieved by proper selection of the gear tooth proportions. Manufacturing tolerances on the gears govern the spacing accuracies of the teeth. Thus, to achieve a satisfactory design, it is necessary to specify correct tooth proportions, and, in addition, the tolerances on the tooth elements must be properly specified.

As a general rule, gearing designed in accordance with the standard systems will not have problems of continuity of action or conjugate action. In those cases where it is necessary to depart from the tooth proportions given in the standard systems, the designer should check both the continuity of action and the conjugate action of the resulting gear design.

Continuity of action. As discussed above, all gear tooth contact must take place along the “line of action”. The shape of this line of action is controlled by the shape of the active profile of the gear teeth, and the length of lines of action is controlled by the outside diameters of the gears (see Figure 4.4). In order to provide a smooth continuous flow of power, at least one pair of teeth must be in contact at all
times. This means that during a part of the meshing cycle, two pairs of teeth will be sharing the load. The second pair of teeth must be designed such that they will pick up their share of the load and be prepared to assume the full load before the first pair of teeth goes out of action.

The base pitch \( p_b \) is defined as follows:

\[
p_b = p \cos \phi = \frac{\pi \cos \phi}{P_d}
\]  

where:

1. \( p \) is the circular pitch of gear
2. \( \phi \) is the pressure angle of gear
3. \( P_d \) is the diametral pitch

Thus, either the outside-diameter circles, the operating pressure angle, or the base pitch must be adjusted so that \( ab \) exceeds the base pitch \( p_b \) by from 20 to 40%.

The most general way of checking continuity of action is by calculating the contact ratio. A numerical index of the existence and degree of continuity of action is obtained by dividing the length of the line of action by the base pitch of the teeth (see Figure 4.4). This is called contact ratio, \( m_p \):

\[
m_p = \frac{L_a}{p_b} \geq 1.2
\]  

A spur gear mesh has only a transverse contact ratio, \( m_p \), whereas a helical gear mesh has a transverse contact ratio, \( m_p \), an axial contact ratio, \( m_F \), (face contact ratio), and a total contact ratio, \( m_t \).

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1 Equations (4.1) through (4.6) given dimensionless. For English-system calculations, use inches for all dimensions. For metric-system calculations, use millimeters for all dimensions.
Equations for contact ratio are as follows:

- Spur gears and helical gears

\[
m_p = \frac{\sqrt{\left(\frac{d_o'}{2}\right)^2 - \left(\frac{d_b}{2}\right)^2} + \sqrt{\left(\frac{D_o'}{2}\right)^2 - \left(\frac{D_b}{2}\right)^2}}{p_b} - C' \sin \phi'
\]  

(4.3)

- Internal gears, spur, and helical gears

\[
m_p = \frac{\sqrt{\left(\frac{d_o'}{2}\right)^2 - \left(\frac{d_b}{2}\right)^2} + \sqrt{\left(\frac{D_o'}{2}\right)^2 - \left(\frac{D_b}{2}\right)^2}}{p_b} - C' \sin \phi'
\]  

(4.4)

- Helical gears, axial contact ratio

\[
m_F = \frac{F \tan \psi}{p}
\]  

(4.5)

- Helical gears, total contact ratio

\[m_t = m_p + m_F\]  

(4.6)

where:

\(d_o'\) – is the outside diameter (effective) of pinion
\(D_o'\) – is the effective outside diameter of gear (diameter to intersection of tip round and active profile); see the following discussion
\(D_i'\) – is the inside diameter (effective), internal gear
\(d_b\) – is the base diameter of pinion
\(D_b\) – is the base diameter of gear
\(D_{bi}\) – is the base diameter of internal gear
\(C'\) – is the operating center-distance of pair
\(\phi'\) – is the operating pressure angle
\(p\) – is the circular pitch (in plane of rotation)
\(p_b\) – is the base pitch
\(F\) – is the length of tooth, axial, the face width of the gear
\(\psi\) – is the helix angle of helical gears

Notes: To achieve correct answers on contact ratio, the following points should be observed.

- The effective outside diameter \(d_o'\) or \(D_o'\) is actually the diameter to the beginning of the tip round, usually \(D_o' = D_o - 2\) (edge round specification) (max). This value rather than the drawing outside diameter should be used since in many cases manufacturing practices for removing burrs produce a large radius, particularly in fine-pitch gears. Thus, a considerable percent of the addendum may not be effective. If the teeth are given a very heavy profile modification, consideration should be given to performing the calculation under (a) full load, assuming contact to the tip of the active profile, and (b) light load, assuming contact near the start of modification. In this case \(d_o'\) or \(D_o'\) is selected
to have a value close to the diameter at the start of modification. This will give an index to the smoothness of operation at these conditions.

- This equation also assumes that the form diameter is a larger value than the undercut diameter. If not, use the value of undercut diameter.
- On occasion, the outside diameter of one or both members are so large relative to the center-distance and operating pressure angle that the tip extends below the base circle when it is tangent to the line of action. Since no involute action can take place below the base circle, the value \( C \sin \phi \) (the total length of the line of action) should be substituted in the equation in place of the value \( \sqrt{\left(d'_o/2\right)^2 - \left(d_o/2\right)^2} \) or \( \sqrt{\left(D'_o/2\right)^2 - \left(D_o/2\right)^2} \) in the case that one or both become larger than the value of \( C \sin \phi \).

**Conjugate action.** Gear teeth are a series of cam surfaces that act on similar surfaces of the mating gear to impart a driving motion. Curves that act on each other with a resulting smooth driving action and with a constant driving ratio are called conjugate curves. The fundamental requirements governing the shapes that any pair of these curves must have are summarized in Willis’ “basic law of gearing” (1841), which states:

**Basic law of gearing:** Normals to the profiles of mating teeth must, at all points of contact, pass through a fixed point located on the line of centers.2

This law of gearing was known to L. Euler, and to F. Savary. Nowadays, this law of gearing is commonly referred to as “Camus-Euler-Savary fundamental law of parallel-axes gearing”, or as “CES–fundamental law of parallel-axes gearing”, for simplicity.

In the case of spur- and helical-type gears, the curves used almost exclusively are those of the involute family. In this type of curves, the fixed point mentioned in the basic law is the pitch point. Since all contact takes place along the line of action, and since the line of action is normal to both the driving and driven involutes at all possible points of contact, and, lastly, since the line of action passes through the pitch point, it can be seen that the involute satisfies all requirements of the basic law of gearing.

Worm gearing, like bevel gearing, is non-involute. The tooth form of worm gearing is usually based on the shape of the worm; that is, the teeth of the worm gear are made conjugate to the worm. In general, worms can be chased, as on a lathe, or cut by such process as milling or hobbing, or can be ground. Each process, however, produces a different shape of worm thread and generally requires a different shape of worm gear tooth in order to run properly.

In the case of face gearing, the pinion member is a spur or helical gear of involute form, but the gear tooth is a special profile conjugate only to the specific pinion. Thus, pinions having a number of teeth larger or smaller than the number for which the face gear was designed will not run properly with the face gear.

**Pitch diameter.** Although pitch circles are not the fundamental circles on gears, they are traditionally the starting point on most tooth designs. The pitch circle is related to the base circle, which is the fundamental circle, by the relationships that follow. Some authorities list as many as nine distinct definitions of pitch circles. The following definitions cover the pitch circles considered in this chapter.

**Standard pitch diameter.** The diameters of the circle on a gear determined by dividing the number of teeth in the gear by the diametral pitch.

The diametral pitch is that of the basic rack defining the pitch and pressure angle of the gear:

\[
D = \frac{N}{P_d}
\]

---

where:
- \( D \) – is the diameter of standard pitch circle
- \( P_d \) – is the diametral pitch of basic rack
- \( N \) – is the number of teeth in gear

**Operating pitch diameter.** The diameter of the circle on a gear, which is proportional to the gear ratio, and to the actual center-distance, at which the gear pair will operate.

A gear does not have an operating pitch diameter until it is meshed with a mating gear. The equations for operating pitch diameter are:

- **External spur and helical gearing**
  
  \[
  d' = \frac{2 C'}{m_g + 1} \text{ pinion member} \tag{4.8}
  \]

  \[
  D' = \frac{2 C'm_g}{m_g + 1} \text{ gear member} \tag{4.9}
  \]

- **Internal spur and helical gearing**
  
  \[
  d' = \frac{2 C'}{m_g - 1} \text{ pinion member} \tag{4.10}
  \]

  \[
  D' = \frac{2 C'm_g}{m_g - 1} \text{ gear member} \tag{4.11}
  \]

where:
- \( d' \) – is the operating pitch diameter of pinion
- \( D' \) – is the operating pitch diameter of gear
- \( C' \) – is the operating center-distance of mesh
- \( m_g \) – is the gear ratio

The operating and the standard pitch circles will be the same for gears operated on center-distances that are exactly standard. The distinction to be made usually involves tolerances on the gear center-distance. Most practical gear designs involve center-distance tolerances that are the accumulated effects of machining tolerances on the center bores and tolerances in bearings (clearances, runout of outer races, and so forth). Thus, gears all operate with maximum and minimum operating pitch diameters.

In Figure 4.5, a pair of gears designed to operate on enlarged center-distances is shown. Both the standard and operating pitch circles are shown. It will be noted that the pitch circles are related to the base circle as follows:

\[
D = \frac{D_b}{\cos \phi} \text{ standard pitch diameter} \tag{4.12}
\]

---

3 Eq. (4.8) through (4.13) should use inches for English calculations, and millimeters for metric calculations.
It will also be noted that the standard pitch circles do not contact each other by the amount that the center-distance has been increased from standard.

In worm gearing, it is convenient to use pitch circles. In this case, however, it is common practice to make the pitch circle of the gear go through the teeth at a diameter at which the tooth thickness is equal to the space width. In the case of the worm, the pitch circle also defines a cylinder at which the width of the threads and spaces are equal. It is also good practice to modify the worm teeth slightly to achieve the required backlash. When this is done, the space widths are greater than the thread thicknesses, as measured on the “standard” pitch cylinder, by the amount of backlash introduced. The pitch cylinder also defines the diameter at which the lead angle, as well as the pressure angle, is to be measured.

**Zones in which involute gear teeth exist.** Although many gear designs utilize “standard” or “equal-dedendum” tooth proportions, it is not always necessary or even desirable to use these proportions. One of the outstanding features of the involute tooth profile is the opportunity it affords for the use of different amounts of addendum and tooth thicknesses on gears of any given pitch and numbers of teeth. These variations can be produced with standard gear tooth generating tools. It is not necessary to buy different cutting or checking tooling for each new value of tooth thicknesses or addendum in the proper tooth thicknesses-addendum relationship in maintained in the design.
As discussed in other chapters of this handbook, the limits of true involute action are established by
the length of the line of action. It has been shown that the end limits defining the maximum usable
portion of the line of action are fixed by the points at which this line becomes tangent to the basic circles.
These limits, $a$ and $b$, are shown in Figure 4.6,a. The largest pinion or gear that will have correct gear
tooth action is defined by circles that pass through points $a$ and $b$ (see Figure 4.6,b). Therefore, any gear
teeth that lie fully within the crosshatched area on Figure 4.6,b will have correct involute action on any
portions of the teeth that are not undercut. Undercut limitations are discussed more fully under “undercut” below. One should not infer that the largest usable outside diameters will be used simultaneously
on both members on a given gear design.

The equations for maximum usable outside diameter are:

$$D_{oG} = 2\sqrt{(C' \sin \phi')^2 + \left(\frac{D_G}{2} \cos \phi\right)^2} \quad \text{max} \quad (4.14)$$

$$D_{oP} = 2\sqrt{(C' \sin \phi')^2 + \left(\frac{D_P}{2} \cos \phi\right)^2} \quad \text{min} \quad (4.15)$$

where:

$C'$ – is the operating center-distance

$\phi'$ – is the cutting or standard pressure angle

$\phi$ – is the operating pressure angle

$D_G$ – is the pitch diameter of gear

$D_P$ – is the pitch diameter of pinion

Operating pitch circles for the gear ratio (2:1) and center-distance chosen for this illustration are
shown in place in Figure 4.6,c. If this center-distance is “standard” for the number of teeth and pitch, the
operating pitch circles shown also be the “standard” pitch circles.

To transmit uniform angular motion, a series of equally spaced involute curves are arranged to act on
each other. These are shown in place in Figure 4.6,d. On both the pinion and the gear member, the
involute curves originate at the base circles and theoretically can go on forever. The more practical
lengths are suggested by solid lines. The spacing of the involutes of both members measured on the base
circles must be equal, and the interval chosen is called the base pitch. The base pitch of the gear or pinion
times the number of teeth in the member must exactly equal the circumference of the base circles.

If similar involute curves of opposite hand are drawn for both base circles, the familiar gear teeth are
achieved. It is customary to measure the distance from one involute curve to the next along the standard
pitch circle. This distance is called the circular pitch. It is also customary to make “standard” tooth
proportions with tooth thicknesses equal to one-half the circular pitch. The standard addendum used for
the gearing system shown in this chapter is equal to the circular pitch divided by $\pi$.

Figure 4.6,e shows “standard” teeth developed on the base circles of Figure 4.6,a through
Figure 4.6,d. It will be noted that the addendum of the 12-tooth pinion is equal to that of the gear and that
the outside diameters Thus, established do not reach out to the maximum values established by the line-
of-action limits.

In modern gear-cutting practice, the tooth thickness of gears as measured on the “standard” pitch
circle is established by the depth to which the generating-type cutter (usually hob or shaper cutter) is fed,
relative to the “standard” pitch diameter. In order to obtain a correct whole depth of tooth, the outside

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4 Equations (4.14) through (4.18) should use inches for English calculations, and millimeters for metric calculations.
FIGURE 4.6  Study of Involute Tooth Development: (a) Maximum Usable Length of Line of Action; (b) Maximum Zone in Which Conjugate Action Can Take Place; (c) Operating Pitch Circles; (d) Development of Involute Profiles; (e) Standard Addendum Tooth Proportions; (f) Short- (Pinion) Addendum Tooth Proportions; (g) Long- (Pinion) Addendum (Abnormal) Tooth Proportions; (h) Long- (Pinion) Addendum (Normal) Tooth Proportions.
diameters of the gear blanks are made larger or smaller than “standard” by twice the amount that the cutter will be fed in or held out relative to the normal or standard amount that would be used for standard gears. If the cutter is to be held out a distance $\Delta c$, the outside diameter of the blank is made $2 \Delta c$ larger than standard and the resultant gear is said to be long addendum. Figure 4.6,f shows an example of a gear in which the cutter was held out sufficiently so that the outside diameter of the gear was equal to the diameter of the maximum usable outside-diameter circle (see also Figure 4.6,b). This is called a long addendum gear. The cutter was fed into the pinion an equal amount, making it short addendum. Reasons for making pinions short addendum are discussed under “Speed increasing drive,” and the effects of the undercutting produced are discussed under “Undercut.”

In order to avoid undercut, or to achieve a more equal balance in tooth strength, it is customary to make the pinion addendum long and that of the gear short. If an attempt is made to design a pinion with the maximum usable outside diameter as established by the line of action (see Figure 4.6,b), difficulties may arise. In the example shown having 12 and 24 teeth, it is not possible to generate a pinion having such an outside diameter.

In Figure 4.6,g is shown the tooth form resulting from such an attempt. The pinion blank was turned to the maximum usable outside diameter, and the cutter was held out an equivalent distance $\Delta c$. As a result of the generating action, the sides of the teeth are involute curves “crossed over” at a diameter smaller than the desired outside diameter. Thus, the teeth are pointed at the outside diameter and also the whole depth is less than anticipated. This effect is less serious in gears having large number of teeth.

The gear tooth that results from this extreme modification is badly undercut. This effect would not have been so great if the gear had more teeth.

The amount of long and short addendum that may be applied to each member of a gear mesh is limited by the three following considerations:

- The length of the usable portion of the line of action will form a maximum limit on the outside diameter of a gear. Diameters in excess of this will not provide additional tooth contact area, since there is no involute portion on the mating gear that can contact this area.
- The diameter at which the teeth become pointed limits the actual or effective outside diameter.
- Undercutting may limit the short-addendum gear. The undercut diameter should be always less than the form diameter.

These three considerations show the extreme limits that bound gear tooth modifications. The designer should not infer that it is necessary to approach these limits in any given gear design. In the treatment of gear tooth modifications, some reasons for making given tooth modifications are considered. In each case, however, the designer must be sure that the amount actually used does not exceed the limits discussed here.

**Pointed teeth.** The previous sections have shown how gear teeth generated by a specific basic rack can have different tooth thicknesses, and that the outside diameters of such gears are altered from standard as a function of the change in tooth thickness. In practice, these tooth profiles are achieved by “feeding in” or “holding out” the gear-generating tools. It is customary to feed a specific cutter to a definite depth into the gear blank which has a greater than standard outside diameter. The cutter, when working at its full cutting depth, will still be held out from the standard $N/P_d$ pitch circle.

Teeth generated thicker than standard will have tips of a width less than standard, since the cutter must be “held out”. For any given number of teeth, the tooth thickness can be increased such that the tip will become pointed at the outside-diameter circle. In Figure 4.7, four gears all having the same number of teeth, diametral pitch, and pressure angle are shown. In Figure 4.7a, the cutter shown by a rack has been fed to the standard depth. The outside diameter of this gear is standard, $(N + 2) P_d$. Note the tooth thickness at the tip. In Figure 4.7b, the outside diameter was somewhat enlarger and the cutter fed in the standard whole-depth distance starting from the enlarged outside diameter. This results in the thicker tooth (which would operate correctly with a standard gear on an enlarged center distance) and a
somewhat thinner tip. In Figure 4.7c, the outside diameter has been established at a maximum value at which it is still possible to achieve a standard whole-depth tooth; the tooth Thus, generated is pointed. Figure 4.7d shows what happens if the maximum outside diameter and tooth thickness are exceeded. The resulting tooth does not have the correct whole depth because the involute curves cross over below the expected outside diameter. This tooth is similar to the one shown on the pinion member in Figure 4.6g.

The amount that the outside diameter of a gear is to be modified is usually a function of the tooth thickness desired. (4.18) below gives the relationship usually employed.

The maximum amount that the tooth thickness of a gear can be increased over standard to just achieve a pointed tooth can be calculated from the simultaneous solution of (4.16) and (4.17):

\[ \Delta D_o^{\text{max}} = N \left( \frac{\cos \phi}{\cos \phi''} - 1 \right) - 2 \]  

(4.16)

---

Equations (4.16) and (4.17) are solved by assuming a series of values for \( \phi'' \). Curves are plotted for \( \phi'' \) vs. \( \phi'' \). The point where the curves cross indicates a simultaneous solution for the two equations. For instance, a 20-tooth pinion of 1 pitch having a \( \phi'' \) pressure angle and cut with a standard cutter has a crossing point for \( \phi'' \) of \( \phi'' \) and \( \phi'' \) of 2.44 in. It should be kept in mind that these equations solve problems for one pitch only (divided by pitch to adjust answer for other pitches) and make no allowance for backlash. In the metric system the solution is in millimeters and for one module. (For other modules, multiply the answer by the module).
\[ \Delta D_{o}^{\text{max}} = \frac{N \left( \text{inv} \phi'' - \text{inv} \phi \right) - \frac{\pi}{2}}{\tan \phi} \]  

(4.17)

The equivalent amount that the tooth thickness must be increased above the standard to achieve this increase in outside diameter is given by (4.18):

\[ \Delta T = d \left( \text{inv} \phi'' - \text{inv} \phi \right) - T \]  

(4.18)

where \( D_{o}^{\text{max}} \) is the maximum outside diameter, at which a tooth having full working depth will come to a point, as follows:

\[ D_{o}^{\text{max}} = \frac{N + 2}{P} + \Delta D_{o} \]  

(4.19)

where:
- \( T \) is the tooth thickness at standard pitch diameter
- \( \phi \) is the standard pressure angle of hob or rack
- \( \phi'' \) is the pressure angle at tip of tooth

**Undercut.** An undercut tooth is one in which a portion of the profile in the active zone has been removed by secondary cutting action. Under certain conditions, the path swept out by the tip of a generating-type cutter will intersect the involute active profile at a diameter greater than the limit diameter. Such a tooth is said to be undercut since no contact with the mating gear can take place between the undercut and the limit circles.

The amount of undercut will depend on the type of tool used to cut the gears. Form-type cutters do not normally produce undercut. In calculating the undercut diameter, it is customary to use an equation based on the type of tooling that will produce the greatest undercut, usually a hob. This is done in order to give the shop the greatest freedom of choice in selecting tools. If the most adverse choice will produce a satisfactory tooth profile, then all other types of cutter that might be chosen by the shop would prove to be satisfactory, except in those cases where undercut is intended to provide clearance for the tips of the mating teeth.

In general, spur gears of 20° pressure angle having 18 or more teeth and made to standard- or long-addendum tooth proportions will not be undercut. In each of the “standard systems” for different types of gear, the minimum number of teeth recommended for each pressure angle is usually based on undercut.

Helical gears can usually be made with fewer tooth than can spur gears without getting into problems of undercut.

Figure 4.8 shows three gears having the same number of teeth, each produced by generating-type cutters. Figure 4.8a is the tooth form of a gear having standard addendum. Figure 4.8b shows the same shape of the tooth that results when the tip of the rack-type cutter is operating at a depth exactly passing through the point of intersection of the line of action and the base circle. Figure 4.8c shows the shape of a tooth with results when the cutter works at a depth somewhat below the intersection of the line of action and the base circle.

Undercutting is product of generating-type cutters. Gears cut by forming-type cutters do not usually have undercut. In certain cases, undercut may ensure tip clearance in mating gears. Racks operating with gears having small number of teeth may show tip interference unless the pinions were generated with special shaper-type cutters or hobs.
4.1.3 LONG- AND SHORT-ADDENDUM GEAR DESIGN

The previous article shows the geometric limitations on the amount that gears can be made long or short addendum. This section indicates cases in which long and short addendum should be considered.

Modification of the addendum of the pinion, and in most cases the gear number, is recommended for gears serving the following applications:

- Meshes in which the pinion has a small number of teeth
- Meshes operating on non-standard center-distances because of limitations on ratio or center-distances
- Meshes of speed-increasing drives
- Meshes designed to carry maximum power for the given weight allowance. (This type of gearing is usually designed to achieve the best balance in strength, wear, specific sliding, pitting, or scoring)
- Meshes in which an absolute minimum of energy loss through friction is to be achieved

*Addendum modification for gears having small number of teeth.* Undercutting is one of the most serious problems occurring in gearing having small numbers of teeth. The amount that gears with small numbers of teeth should be enlarged (made long addendum) to avoid undercut, has been standardized.

![Diagram of gear tooth design](image-url)

**FIGURE 4.8** Cutting Short-Addendum Teeth: (a) Standard Tooth; (b) Short-Addendum Tooth; (c) Short-Addendum (Abnormal) Tooth.
The values of modification are based on the use of a hob or rack-type cutter and as a result are more than adequate for gears cut with circular, shaper-type cutters. The values of addendum modification recommended for each number of teeth are shown in Table 4.1.

When two gears, each containing a small number of teeth, must be operated together, it may be necessary to make both members long addendum to avoid undercut. Such teeth will have a tooth thickness which is larger than standard and which will necessitate the use of greater-than-standard center-distance for the pair.

Modifications to avoid undercut fall into three categories:

- **If both members have fewer teeth than the number critical to avoid undercut, increase the center-distance so that the operating pressure angle is increased.** Then get appropriate values of the addendum and whole depth for the pinion and for the gear.

- **If the pinion member has fewer teeth than the critical number, and the mating gear has considerably more, the usual practice is to decrease the addendum of the gear by the amount proportional to the amount that the pinion is increased.** This results in a pair of gears free from undercut that will operate on a standard center-distance.

- **If the pinion member has fewer teeth than the critical number, and the gear just slightly more,** a combination of the first two practices above may be employed.

An alternate is to increase the pinion addendum by the required amount and increase the center-distance by an equivalent amount to make it possible to use a standard gear.

**Speed-increasing drives.** Most gear trains are speed reducing (torque increasing), and most data on gear tooth proportions are based on the requirements of this type of gear application. The kinematics of speed-increasing drives is somewhat different, and as a result, special tooth proportions should be considered for this type of gear application. As in the case of conventional drives, the problems to be discussed here are most serious in meshes involving small number of teeth.

The first of these problems involves the tendency of the tip edge of the pinion tooth to gouge into the flank of the driving gear tooth. This gouging can come about as a result of spacing errors in the teeth of either member which allow the flank of the gear tooth to arrive at the theoretical contact point on the line of action before the pinion does. The pinion tooth has to deflect to get into the right position or it will gouge off a sliver of gear tooth side. If the gears are highly loaded, the unloaded pinion tooth entering the mesh will be out of position (lagging) since it is not deflected. The gear tooth is in effect slightly ahead of where it should be. The result is the same as if the gear tooth had an angular position error. The bearing and lubricating problems at the beginning point of contact are particularly bad. The edge of the pinion tooth tends to act as a scraper and remove any lubricating film that may be present, for some distance along the flank of the tooth.

One possible solution to this problem is to give the tip of the pinion tooth a moderate amount of tip relief. This provides a sort of sled-runner condition which is easier to lubricate and which helps the pinion tooth find the proper position relative to the gear tooth with less impact.

A better solution, which may also be combined with the tip modification, is to modify the tooth thickness and addendums so as to get as much of the gear tooth contact zone in the arc of recess as possible.

**Power drives (optimal design).** As shown in other chapters, gears fail in one or more of the following ways: actual breaking of the teeth, pitting, scoring, or by wear. In the case of drives with gears of standard tooth proportions and similar metallurgy, the weakest member is the pinion, and if tooth breakage does occur, it is generally in the pinion. This is a result of the weaker shape of the pinion tooth, as well as the larger number of fatigue cycles that it accumulates. This problem can be relieved to a considerable degree by making the pinion somewhat longer and, in so doing, increasing the thickness of the teeth and also improving their shapes. If a standard center-distance is to be maintained, the gear addendum is reduced a proportional amount. If the proper values are chosen, the pinion tooth strength
### TABLE 4.1
**Values of Addendum**

<table>
<thead>
<tr>
<th>No. of Teeth in Pinion</th>
<th>Coarse-Pitch Teeth (1 through 19 ( P_d )), 20°</th>
<th>Coarse-Pitch Teeth (1 through 19 ( P_d )), 25°</th>
<th>Fine-Pitch Teeth (20 ( P_d ) and finer), 20°</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \alpha_p ) Pinion</td>
<td>( \alpha_G ) Gear</td>
<td>Recommended Min. No. of Teeth</td>
</tr>
<tr>
<td>7</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>8</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>9</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>10</td>
<td>1.468</td>
<td>0.532</td>
<td>25</td>
</tr>
<tr>
<td>11</td>
<td>1.409</td>
<td>0.591</td>
<td>24</td>
</tr>
<tr>
<td>12</td>
<td>1.351</td>
<td>0.649</td>
<td>23</td>
</tr>
<tr>
<td>13</td>
<td>1.292</td>
<td>0.708</td>
<td>22</td>
</tr>
<tr>
<td>14</td>
<td>1.234</td>
<td>0.766</td>
<td>21</td>
</tr>
<tr>
<td>15</td>
<td>1.175</td>
<td>0.825</td>
<td>20</td>
</tr>
<tr>
<td>16</td>
<td>1.117</td>
<td>0.883</td>
<td>19</td>
</tr>
<tr>
<td>17</td>
<td>1.058</td>
<td>0.942</td>
<td>18</td>
</tr>
<tr>
<td>18</td>
<td>1.000</td>
<td>1.000</td>
<td>...</td>
</tr>
</tbody>
</table>

**Note:** The values in this table are for gears of 1 diametral pitch. For other sizes divide by the required diametral pitch.

The values of addendum shown are the minimum increase necessary to avoid undercut. Additional addendum can be provided for special applications to balance strength. See Table 4.2.

*These values are less than the proportional amount that the tooth thickness is increased (see Table 4.15) in order to provide a reasonable top land.
will be increased and the gear tooth strength somewhat reduced, which will result in almost equal gear and pinion tooth strength. This will result in an overall increase in the strength of the gear pair.

Several authorities have suggested addendum modifications which will balance scoring, specific sliding, and tooth strength. Unfortunately, each balance results in different tooth proportions so that the designer has to use proportions that will balance only one feature or else proportions that are a compromise. Table 4.2 gives values that are such a compromise.

Experimental data seem to indicate that a pair that is corrected to the degree that seems to be indicated by tooth layouts or by calculation for balanced tooth strength will usually result in an overcorrection to the pinion member. The gear is not as strong as form factors seem to indicate. Notch sensitivity in higher hardness ranges seems to be a problem, especially if the gear is to experience a great number of cycles of loading.

In small numbers of teeth, the correction required to avoid undercut on gears operated on standard center-distances is excessive, in many cases, in respect to equal tooth strength. An overcorrection in pinion tooth thickness can lead to an excessive tendency to score. As a result, the values of addendum recommended in Table 4.2 represent a compromise among balanced strength, sliding, and scoring.

### Table 4.2

**Values of Addendum for Balance Strength**

<table>
<thead>
<tr>
<th>$m_G$, (Gear Ratio) $N_G/N_P$</th>
<th>$a$ (Addendum)</th>
<th>$m_G$, (Gear Ratio) $N_G/N_P$</th>
<th>$a$ (Addendum)</th>
</tr>
</thead>
<tbody>
<tr>
<td>From</td>
<td>To</td>
<td>Pinion, $a_P$</td>
<td>Gear, $a_G$</td>
</tr>
<tr>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>1.001</td>
<td>1.020</td>
<td>1.010</td>
<td>0.990</td>
</tr>
<tr>
<td>1.021</td>
<td>1.030</td>
<td>1.020</td>
<td>0.980</td>
</tr>
<tr>
<td>1.031</td>
<td>1.040</td>
<td>1.030</td>
<td>0.970</td>
</tr>
<tr>
<td>1.041</td>
<td>1.050</td>
<td>1.040</td>
<td>0.960</td>
</tr>
<tr>
<td>1.051</td>
<td>1.060</td>
<td>1.050</td>
<td>0.950</td>
</tr>
<tr>
<td>1.061</td>
<td>1.080</td>
<td>1.060</td>
<td>0.940</td>
</tr>
<tr>
<td>1.081</td>
<td>1.090</td>
<td>1.070</td>
<td>0.930</td>
</tr>
<tr>
<td>1.091</td>
<td>1.110</td>
<td>1.080</td>
<td>0.920</td>
</tr>
<tr>
<td>1.111</td>
<td>1.120</td>
<td>1.090</td>
<td>0.910</td>
</tr>
<tr>
<td>1.121</td>
<td>1.140</td>
<td>1.100</td>
<td>0.900</td>
</tr>
<tr>
<td>1.141</td>
<td>1.150</td>
<td>1.110</td>
<td>0.890</td>
</tr>
<tr>
<td>1.150</td>
<td>1.170</td>
<td>1.120</td>
<td>0.880</td>
</tr>
<tr>
<td>1.170</td>
<td>1.190</td>
<td>1.130</td>
<td>0.870</td>
</tr>
<tr>
<td>1.190</td>
<td>1.210</td>
<td>1.140</td>
<td>0.860</td>
</tr>
<tr>
<td>1.210</td>
<td>1.230</td>
<td>1.150</td>
<td>0.850</td>
</tr>
<tr>
<td>1.231</td>
<td>1.250</td>
<td>1.160</td>
<td>0.840</td>
</tr>
<tr>
<td>1.251</td>
<td>1.270</td>
<td>1.170</td>
<td>0.830</td>
</tr>
<tr>
<td>1.271</td>
<td>1.290</td>
<td>1.180</td>
<td>0.820</td>
</tr>
<tr>
<td>1.291</td>
<td>1.310</td>
<td>1.190</td>
<td>0.810</td>
</tr>
<tr>
<td>1.311</td>
<td>1.330</td>
<td>1.200</td>
<td>0.800</td>
</tr>
<tr>
<td>1.331</td>
<td>1.360</td>
<td>1.210</td>
<td>0.790</td>
</tr>
<tr>
<td>1.361</td>
<td>1.390</td>
<td>1.220</td>
<td>0.780</td>
</tr>
<tr>
<td>1.391</td>
<td>1.420</td>
<td>1.230</td>
<td>0.770</td>
</tr>
</tbody>
</table>

*Note: Do not select values from this table for the pinion member that are smaller than those given in Table 4.1.*
Gears with teeth finer than about 20 diametral pitch (generally) cannot score, since the tooth is not strong enough to support a scoring load; therefore, the values for addendum increase in fine-pitch gears are somewhat larger than the values for coarse-pitch power gearing.

*Low-friction gearing.* In cases where a speed-increasing gear train is to transmit power or motion with the least possible loss of energy, the selection of the tooth proportions is of considerable importance. The sliding should be kept as low as possible, and as much of the tooth action should be put into the arc of recess as possible.

Figure 4.9 shows two involute curves (tooth profiles) in contact at two different points along the line of action. The direction in which the driven pinion tooth slides along the driving gear tooth is shown by the arrows. This example is a speed-increasing drive which is the most sensitive to friction between the teeth. At the pitch point (where the line of centers crosses the line of action) there is no sliding, and it is at this point that the direction of relative sliding of one tooth on the other changes.

The forces shown in Figure 4.9 are those acting on the driven pinion. The subscripts “a” are the values considered in the arc of approach and “r” are those considered in the arc of recess. The normal driving force $W_N$ is the force that occurs at the pitch point, if there were no friction at the point of gear tooth contact, would be the force at all other points of contact along the line of action. Since there is friction, the friction vectors $f_a$ and $f_r$ oppose the sliding of the gear teeth in the arcs of approach and recess. Note the change in direction due to the change in direction of sliding. The angle of friction is $\Phi$ and is assumed

![Figure 4.9 Effect of Friction on Tooth Reactions.](image-url)
to be the same in both cases. The torque exerted by the shaft driving the driving gear \( T_{DR} \) manifests itself in arc of approach as:

\[
T_{DR, a} = W_N \times R_{NG} \quad \text{if no friction}
\]

\[
T_{DR, a} = W_f \times R_{fG_a} \quad \text{if friction is assumed}
\]

and in the arc of approach as:

\[
T_{DR, a} = W_N \times R_{NG} \quad \text{if no friction}
\]

\[
T_{DR, a} = W_f \times R_{fG_a} \quad \text{if friction is assumed}
\]

The resisting moments are, in the arc of approach:

\[
T_{DN, a} = W_N \times R_{NP} \quad \text{if no friction}
\]

\[
T_{DN, a} = W_f \times R_{fP_a} \quad \text{if friction is assumed}
\]

and the corresponding moments are, in the arc of recess:

\[
T_{DN, r} = W_N \times R_{NP} \quad \text{if no friction}
\]

\[
T_{DN, r} = W_f \times R_{fP_r} \quad \text{if friction is assumed}
\]

Note that, in all cases above, single tooth contact is assumed.

\textit{Efficiency} is output divided by input and in this case is the torque that would appear on the driven shaft when friction losses are considered, compared with the torque that would result if no losses occurred.

(4.28) below shows the efficiency of the mesh (single tooth contact) for the contacts occurring in the arc of approach, and (4.29) shows the efficiency in the arc of recess:\(^6\)

\[
E_{\text{approach}} = \frac{R_f}{R_f \times R_{NP}} \cdot \frac{R_{NG}}{R_{NP}}
\]

\[
E_{\text{recess}} = \frac{R_f}{R_f \times R_{NP}} \cdot \frac{R_{NG}}{R_{NP}}
\]

In the case of speed-increasing drives, the increase efficiency can have considerable significance; cases have occurred in which, for every high ratios and poor lubrication, the speed increaser actually became self-locking.

\(^6\) Equations (4.28) and (4.29) can be used in the metric system by using \textit{newtons} for force and \textit{millimeters} for distance. This makes the units of torque \( \phi^{''} \).
4.1.4 Special Design Considerations

Several special considerations should be kept in mind during the evolution of a gear design.

**Interchangeability.** Two types of interchangeability are related to gearing.

The first, and most generally recognized, is part interchangeability. This means that, if a part made to a specific drawing is damaged, a similar part made to the same drawing can be put in its place and can be expected to perform exactly the same quality of service. In order to achieve this kind of interchangeability, all parts must be held to carefully selected tolerances during manufacture, and the geometry of the parts must be clearly defined.

The second kind is engineering interchangeability. This type provides the means whereby a large variety of different sizes or number of teeth can be produced with a very limited number of standard tools. Parts so made may or may not be designed to have part interchangeability. The several systems of gear tooth proportions are based on engineering interchangeability. By careful design and skill in the application of each system, it is possible to design gears that will perform in almost any application and that can be made with standard tools. Only in the most exceptional cases will special tools be required. The advantages of gearing designed with engineering interchangeability are the lower tool costs and the ease with which replacement or alternate gears can be designed.

Gears procured through catalogue source are good examples of gears having both part and engineering interchangeability.

**Tooth thickness.** The thickness of gear teeth determines the center-distance at which they will operate, the backlash that they will have, and, as discussed in previous sections, their basic shape. One of the important calculations made during the design of gear teeth is establishment of tooth thickness.

It is essential to specify the distance from the gear axis at which the desired tooth thickness is to exist. The usual convention is to use the distance that is established by the theoretical pitch circle \( N/P \). Thus, if no other distance is shown, the specified tooth thickness is assumed to lie on the standard pitch circle. In certain cases, the designer may wish to specify a thickness, such as the chordal tooth thickness, at a diameter other than the standard pitch diameter. This specified diameter should be clearly defined on the gear drawings.

The actual calculation of tooth thickness is usually accomplished by the following procedure:

1. Theoretical tooth thickness is established.
   a. If the gears are of conventional design and are to be operate on standard center-distance, the tooth thickness used is one-half the circular pitch.
   b. If special center-distances are to be accommodated, the below (4.30) and (4.31) may be used:

\[
\cos \phi_2 = \frac{C}{C_2} \cos \phi
\]  

(4.30)

where:

\( \phi_2 \) - is the operating pressure angle

\( C \) - is the standard center-distance

\( \phi \) - is the standard pressure angle

\( C_2 \) - is the operating center-distance

\[
T_P + T_G = \pi + (\text{inv} \phi_2 - \text{inv} \phi)(N_P + N_G)
\]

(4.31)
where:

- \( T_P \) is the tooth thickness of pinion member (at 1 diametral pitch)
- \( T_G \) is the tooth thickness of gear member (at 1 diametral pitch)
- \( N_P \) is the number of teeth in pinion
- \( N_G \) is the number of teeth in gear

2. After the theoretical tooth thickness is established, the allowance for backlash is made.
   a. Backlash allowance may be shared equally by pinion and gear. In this case the theoretical tooth thickness of each member is reduced by one-half the backlash allowance.
   b. In case of pinions with small numbers of teeth which have been enlarged to avoid problems of undercut, it is customary to take all the backlash allowance on the gear member. This avoids the absurdity of increasing the tooth thickness to avoid undercut and then thinning the teeth to introduce backlash.

   In case (a):
   \[
   T_{\text{actual}} = T_{\text{theoretical}} - \frac{B}{2}
   \]  
   (4.32)

   where \( B \) is backlash allowance.

   In case (b):
   \[
   T_{\text{actual, pinion}} = T_{\text{theoretical}}
   \]  
   (4.33)

   \[
   T_{\text{actual, gear}} = T_{\text{theoretical}} - B
   \]  
   (4.34)

3. The teeth are lastly given an allowance for machining tolerance. This tolerance gives the machine operator a size or processing tolerance. Usually this is a unilateral tolerance.

It is beyond the scope of this general-purpose book on gear technology to go deep into gear design details.

**Tooth profile modifications.** Errors of manufacture, deflections of mountings under load, and deflections of the teeth under load all combine to prevent the attainment of true involute contact in gear meshes. As a result, the teeth do not perform as they are assumed to by theory. Premature contact at the tips or excessive contact pressures at the ends of the teeth give rise to noise and/or gear failures. In order to reduce these causes of excessive tooth loads, profile modification is a usually practice. Remember: Only base pitch preserving modification of the tooth profile is allowed.

**Transverse of profile modification.** It is customary to consider the shape of the individual tooth profile. Actually, an operating gear presents to the mating gear tooth profiles whose shapes are distorted by errors in the profile shape (involute) and in tooth spacing, as well as by deflections due to contact loads acting at various places along the tooth. Ideally, teeth under load should appear to the mating gear

---

\(^7\) The best way to use this equation is to start with \( C \) being the center-distance for 1 diametral pitch (or for 1 module in metric dimensions). The \( C_2 \) is computed by Eq. (4.30). The tooth thicknesses determined are not at the operating pitch diameters but are at the 1 diametral pitch center-distance of \( C \).
to have perfect involute profiles and to have perfect spacing. This would require a tooth profile when subjected to a load. Since loads vary, this is not practical. Usual practice is to give tip or root profile modifications or otherwise correct involute profiles that will be distorted by contact loads.

Tip modification usually takes the form of a sliding thinning of the tip of the tooth, starting at a point about halfway up the addendum. The amount of this modification is based on the probable accumulated effects of the following.

**Allowances for errors of gear manufacture.** The correct involute profile may not be achieved as a result of manufacturing tolerances in the cutter used to cut the gear teeth or in the machine guiding the cutter. Errors in spacing may also be introduced by the cutter or machine. The result is that the tips of the teeth will attempt to contact the mating gear too soon or too late. Tip modification produces a sort of sled-runner shape to help guide the teeth into full contact with the least impact.

**Allowances for deflection under load.** Although the teeth of a gear may have a correct profile under static conditions, the loads imposed may deflect the teeth in engagement to such a degree that the teeth that are just entering mesh will not be in relative positions that are correct for smooth engagement. These tooth deflections cause two errors: The actual tooth profiles are not truly conjugate under load and therefore do not transmit uniform angular motion, and the spacing, or relative angular placement of the driving teeth relative to the driven teeth is such that smooth tooth engagement cannot take place.

Several methods may be used to compensate for these effects. The most usual is to provide tip relief as discussed above. The flanks of the teeth may also be relieved. Tip and flank relief assume that the normal tooth would tend to contact too soon. If the tips of the driven gear are made slightly thin, the tooth will be able to get into a position on the line of contact before contact between that tooth and its driving mate actually occurs. The sled-runner effect of the tip modification will allow the tooth to assume full contact load gradually.

In the case of spur gearing, the amount of tip relief should be based on the sum of the allowance for probable tooth-to-tooth spacing errors and for assumed deflection of the teeth already in mesh. (4.35) and (4.36) below give good first approximations of the amount of tip relief required. The values obtained by these equations should be modified by experience for the best overall performance.

*Modification at first point of contact:*

\[
\text{Modification} = \frac{\text{driving load (lb)} \times 3.5 \times 10^{-7}}{\text{face width (in.)}}
\]  
(4.35)

Remove stock from tip of driven gear.

*Modification at last point of contact:*

\[
\text{Modification} = \frac{\text{driving load (lb)} \times 2.0 \times 10^{-7}}{\text{face width (in.)}}
\]  
(4.36)

Remove stock from tip of driven gear.

In general, gear teeth that carry a load in excess of 2000 lb per in. of face width for more than 1,000,000 cycles should have modification. Those under 1000 lb per in. of face width do not generally require modification.

Bevel gear teeth are often modified to accommodate mounting misalignments, tooth errors, and deflections due to load. The geometry of bevel gear teeth allows profile modifications both along the length of the teeth and form root to tip. In the case spiral and Zerol bevel gears cut with face mill-type cutters,

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8 Zerol is a trademark registered by the Gleason Works, Rochester, NY.
the contacting faces of the gear and pinion teeth can be easily made to a slightly different radius of
curvature on each member. This is equivalent to crowning of spur gears and is called mismatch.

Tips of the teeth may be given relief, and in some cases the root of the flank is relieved (undercut).
This is most commonly done on spiral bevel gears and hypoid gears that are to be lapped. It is done by
cutters having a special protuberance called Toprem\textsuperscript{9} cutters.

Mismatch is calculated into the machine settings and is therefore beyond the scope of this chapter.

Axial modifications. In general, it is expected that a gear tooth will carry its driving load across the
full face width (if a spur gear).

Because of deflections in shafts, bearings, or mountings, when under load, or because of errors in the
manufacturing of these parts or in the gears, the teeth may not be quite parallel, and end loading may
result. Since a heavy load on the end of a gear tooth will often cause it to break off, attempts to avoid
loading by relieving the ends of the teeth (crowning) are often made. A gear tooth that has been crowned
is slightly thicker at the center section of the tooth than at the ends when measured on the pitch cylinder.

In effect, crowning allows a rocking-chair-like action between the teeth when the shafts deflect into in-
creasingly nonparallel positions. Heavy concentrations of load at the ends of the teeth are avoided. The fact
that the whole face cannot act when the shafts are parallel requires that the load imposed upon a set of crowned
gears be less than could be carried on a similar non-crowned pair if parallel teeth could be maintained.

In general, the ends of crowned gears are made from 0.0005 to 0.002 in. thinner at the ends as
compared with the middle, when measured on the circular-arc tooth thickness.

Spur and helical gearing are crowned by means of special attachments on the gear tooth finishing
machines.

Bevel gearing is crowned, in effect, by the shape of the cutting tools and the way in which they are
driven in relation to the teeth. Coniflex\textsuperscript{10} teeth are straight bevel teeth cut on special generating machines
which produce lengthwise crowning. Spiral and Zerol bevel gears, and hypoid gears are given the effect
of crowing by the use of a different radius of lengthwise curvature on the convex and the concave sides
of the teeth.

Throated worm gears can be given the effect of crowning by the use of slightly oversize hobs.

Face gears are given the effect of crowning by the selection of a cutter having one or more teeth than
the mating gear.

Root fillets. The shape and minimum radius of curvature that the root fillet of a tooth will have
depends on the type and design of the cutting tool used to produce the gear tooth. The shape of the root
fillet, as well as its radius, and the smoothness with which it blends into the root land and active profile
of a gear tooth can have a profound effect on the fatigue strength of the finished gear. The radius of the fillet
at the critical cross section of the tooth is controlled on drawings by specifying the minimum acceptable
fillet radius. The point of minimum radius occurs almost adjacent to the root land in the case of gears cut
by hobs. An equation giving a reasonable evaluation or the minimum radius for teeth cut by hobs is:\textsuperscript{11}

\begin{equation}
r_f = 0.7 \left[ r_T + \frac{(h_t - a - r_T)^2}{d} + h_t - (a + r_T) \right]
\end{equation}

where:

$r_f$– is the minimum calculated fillet radius produced by hobbing or generating grinding

$r_T$– is the edge radius of the generating rack, hob, or grinding wheel

\textsuperscript{9} Toprem is a trademark registered by the Gleason Works, Rochester, NY.
\textsuperscript{10} Coniflex is a trademark registered by the Gleason Works, Rochester, NY.
\textsuperscript{11} (4.37) is valid for the metric system by using millimeters instead of inches.
- $a$ – is the addendum of gear
- $h$ – is the whole depth of gear
- $d$ – is the pitch diameter of gear
- $\psi$ – is the helix angle (use $0^\circ$ for spur gears)

The edge radius specified for the generating tool will, in general, depend on the service the gear is to perform or on special manufacturing considerations. Table 4.3 shows suggested values of edge radius for various gear applications.

Other types of manufacturing tools can be designed to produce the minimum fillet radius as obtained from (4.37). The shape of the fillet will be somewhat different, however.

A common method of checking the minimum radius of the coarser-pitch gears is to lay a pin in the fillet zone and note that the contact is along a single line.

The constant 0.7 is to allow a reasonable working tolerance to the manufacturer of the tools. Edge breakdown of hobs and cutters will tend to increase the radius produced. This is particularly true of hobs and cutters for very-fine-pitch gears.

**Effective outside diameter.** It is customary to consider the outside diameter of a gear as the outer boundary of the active profile of the tooth. In several cases this approximation is not good enough.

- In very-fine-pitch gears that have been burr brushed, the tip round may be quite large in proportion to the size of the teeth even though it is only a few thousands in radius by actual measurement. Since no part of this radius can properly contact the mating tooth, the outside diameter is, from the standpoint of conjugate action, limited to the diameter where the tip radius starts. Effective outside diameter should be used instead of outside diameter in calculations of contact ratio.

- In some gear meshes in which the pinion member contains a small number of teeth, the tips of gear teeth may be found to be extending into the pinion spaces to a depth greater than that bounded by the line of action. In Figure 4.6b the dimension $D_o/2$ is the maximum effective diameter to the pinion. The actual outside diameter of the pinion may exceed this value, however. In calculating contact ratio, the effective diameter as limited by the pinion base circle and center-distance should be used.

**Width of tip of tooth.** The tooth thickness at the tip of the tooth is a convenient index of the quality of a gear design. For most power gearing applications, the thickness of the gear tooth should not be more than $1\frac{1}{2}$ to 2 times that of the pinion tooth at the tip.

If the tooth thickness (arc), at the pitch diameter (standard) is known, the following equation will give its thickness at the tip:

$$t_o = D_o \left( \frac{t}{D} + \text{inv}\phi - \text{inv}\phi_o \right)$$

(4.38)

$$\cos \phi_o = \frac{D \cos \phi}{D_o}$$

(4.39)

where:
- $t_o$ – is the tooth thickness at outside diameter $D_o$
- $D_o$ – is the outside diameter of gear, or diameter where tooth thickness is wanted
- $t$ – is the arc tooth thickness at $D$, or at known diameter
- $D$ – is the standard pitch diameter, or diameter where $t$ is known
- $\phi$ – is the standard pressure angel, or pressure angle where tooth thickness $t$ is known
- $\phi_o$ – is the pressure angle at outside diameter or at diameter where tooth thickness is wanted
TABLE 4.3
Basic Tooth Proportions for Helical Gears

<table>
<thead>
<tr>
<th>Tooth Form No.</th>
<th>Pitch of Teeth</th>
<th>Pressure Angle</th>
<th>Depth of Teeth</th>
<th>Helix Angle</th>
<th>Addendum</th>
<th>Tooth Thickness (arc)</th>
<th>Edge Radius of Generating Rack</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Diametral</td>
<td>Circular</td>
<td>Axial</td>
<td>Normal</td>
<td>Transverse</td>
<td>Normal</td>
<td>Transverse</td>
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<tr>
<td></td>
<td>$P_{ma}$</td>
<td>$P_d$</td>
<td>$P_t$</td>
<td>$p$</td>
<td>$p_n$</td>
<td>$h_k$</td>
<td>$h_t$</td>
</tr>
<tr>
<td></td>
<td>$\phi_a$</td>
<td>$\phi$</td>
<td>$\psi$</td>
<td>$a$</td>
<td>$t$</td>
<td>$r_T$</td>
<td>$r_T$</td>
</tr>
<tr>
<td>1</td>
<td>1.03528</td>
<td>1</td>
<td>3.03454</td>
<td>3.14259</td>
<td>11.72456</td>
<td>2.0000</td>
<td>2.3500</td>
</tr>
<tr>
<td>2</td>
<td>1.08360</td>
<td>1</td>
<td>2.89190</td>
<td>3.14259</td>
<td>7.40113</td>
<td>2.0000</td>
<td>2.3500</td>
</tr>
<tr>
<td>3</td>
<td>1.08360</td>
<td>1</td>
<td>2.89185</td>
<td>3.14259</td>
<td>7.40113</td>
<td>1.8400</td>
<td>2.2000</td>
</tr>
<tr>
<td>4</td>
<td>1.15470</td>
<td>1</td>
<td>2.72070</td>
<td>3.14259</td>
<td>5.44140</td>
<td>1.7400</td>
<td>2.0500</td>
</tr>
<tr>
<td>5</td>
<td>1.22077</td>
<td>1</td>
<td>2.57340</td>
<td>3.14259</td>
<td>4.8666</td>
<td>1.6400</td>
<td>1.9500</td>
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<td>6</td>
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<td>1</td>
<td>2.22144</td>
<td>3.14259</td>
<td>3.14259</td>
<td>1.4200</td>
<td>1.7000</td>
</tr>
<tr>
<td>7</td>
<td>1</td>
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<td>...</td>
<td>...</td>
<td>...</td>
<td>2.0000</td>
<td>2.2500</td>
</tr>
<tr>
<td>8</td>
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<td>...</td>
<td>...</td>
<td>...</td>
<td>2.0000</td>
<td>2.3300</td>
</tr>
<tr>
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<td>2.2700</td>
</tr>
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<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
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<td>2.3500</td>
</tr>
<tr>
<td>11</td>
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<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
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<td>2.4000</td>
</tr>
<tr>
<td>12</td>
<td>1</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>2.0000</td>
<td>2.4500</td>
</tr>
<tr>
<td>13</td>
<td>1</td>
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<td>2.0180</td>
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<td>2.1170</td>
</tr>
<tr>
<td>15</td>
<td>1</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>2.1570</td>
</tr>
<tr>
<td>16</td>
<td>1</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
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<td>2.1550</td>
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<tr>
<td>17</td>
<td>1</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>2.6150</td>
<td>3.1580</td>
</tr>
<tr>
<td>18</td>
<td>1</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>2.0000</td>
<td>2.3500</td>
</tr>
<tr>
<td>19</td>
<td>1</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>2.0000</td>
<td>2.3500</td>
</tr>
<tr>
<td>20</td>
<td>1.15470</td>
<td>1</td>
<td>2.72070</td>
<td>3.14259</td>
<td>5.44140</td>
<td>22°30′00″</td>
<td>25°33′41″</td>
</tr>
<tr>
<td>21</td>
<td>1.10338</td>
<td>1</td>
<td>2.84725</td>
<td>3.14259</td>
<td>6.73717</td>
<td>25°00′00″</td>
<td>27°13′35″</td>
</tr>
</tbody>
</table>
**Pointed tooth diameter.** An independent method of checking the quality of a long- and short-addendum gear design is to calculate the diameter at which the teeth would come to a point. If this value is smaller than the value of outside diameter chosen by other means, the design should be recalculated. Equation (4.40) and (4.41) provide one method of calculating pointed tooth diameters:

\[ \text{inv } \phi_{oP} = \frac{t}{D} + \text{inv } \phi \]  
(4.40)

\[ D_{oP} = \frac{D \cos \phi}{\cos \phi_{oP}} \]  
(4.41)

where \( \phi_{oP} \) equals to pressure angle at pointed tooth diameter, and \( D_{oP} \) is the pointed tooth diameter. See (4.39) for other symbols.

**Purpose for backlash.** In general, backlash is the lost motion between mating gear teeth. It may be measured along the line of action or on the pitch cylinder of the gears (transverse backlash) and, in the case of helical gears, normal to the teeth.

In a set of meshing gears, the backlash that exists is the result of the actual center-distance at which the gears operate, and the thickness of the teeth. Changes in temperature, which may cause differential expansion of the gears and mountings, can produce appreciable changes in backlash.

When establishing the backlash that a set of gears will require, the following should be considered:

- The minimum and maximum center-distance. These values are the result of tolerance buildings in the distance between the bores supporting the bearings on which the gears are mounted as well as the basic or design values. Antifriction bearings, for example, have runout between the bore and the inner ball path. As the shaft and the inner race of the bearing rotate, and the outer race creeps in the housing, the center-distance will vary by the amount of the eccentricities of these bearing elements.
- The thickness of the teeth, as measured at a fixed distance from the center on which the gear rotates, will vary because of gear runout. Also, the workman cutting the gear is given a tooth thickness tolerance to work to, which introduces tooth thickness variations from one gear to the next.

The minimum backlash will occur when all the tolerances react all at the same time to give the shortest center-distance and the thickest teeth with the high points of gear runout. The maximum backlash will occur when all the tolerances move in the opposite directions.

Design backlash is incorporated into the mesh to ensure that contact will not occur on the non-driving sides of the gear teeth. Although backlash may be introduced by increasing the center-distance, it is usually introduced by thinning the teeth. The minimum value should be at least sufficient to accommodate for a lubricating film on the teeth.

Sometimes a statistical approach is used, since there are a sufficient number of tolerances involved; thus, the design backlash introduced may not be numerically as large as the possible adverse buildup of tolerances. This approach is particularly handy in instrument gearing, since the maximum backlash allowable usually has to be held to a minimum.

By definition, backlash cannot exist in a single gear. Backlash is a function of the actual center-distance on which the gears are operated, and the actual thicknesses of the teeth of each gear.

It is customary to use generally recognized values of center-distance tolerance and gear tooth tolerances for power gearing. If this is done, and the calculated tooth thicknesses are reduced by the amounts of design backlash as shown in Table 4.4, satisfactory gears should result. In the case of instrument gearing, either the values in Table 4.4 may be used or the tooth thickness actually required and the resulting maximum backlash may be calculated.
The backlash that is measured in gears under actual operation will in all probability be considerably larger than the values given in Table 4.4, since these values are for design and do not include a correction for normal machining tolerances.

**Backlash: Recommended values.** These dimensions, shown in Figure 4.2, are the distances from the axis of spur, helical, and internal gears at which the active profiles of the teeth begin. Form diameter is the lowest point (spur and helical gearing) at which the mating tooth can contact the active profile (see Figure 4.5).

### 4.2 STANDARD SYSTEMS OF GEAR TOOTH PROPORTIONS

A standard system of gear tooth proportions provides a means of achieving engineering interchangeability for gears of all numbers of teeth of a given pitch and pressure angle. Because of the large variety of tooth proportions that are possible, it has been found desirable to standardize on a limited number of tooth systems. These systems specify the various relationships among tooth thicknesses, addendum, working depth, and pressure angle.

The following sections provide the basic data covered by most of the systems. In each of the following systems, the tooth proportions are shown in terms of the basic rack of that system. In each case, all gears designed to the basic tooth proportions will have engineering interchangeability.

#### 4.2.1 STANDARD SYSTEMS FOR SPUR GEARS

The following data are based on the information contained in standards for “20-Degree Involute Fine-Pitch System for Spur and Helical Gears”, and for “Tooth proportions for Coarse-Pitch Involute Spur Gears”.

**Limitations in use of standard tables.** Caution should be exercised in using the data contained in Table 4.5. The items shown apply only to gears that meet the following requirements:
Standard-addendum gears must exceed the minimum numbers of teeth shown in Table 4.6. Long- and short-addendum designs, as derived from Tables 4.5, 4.7, and 4.8, are to be used for speed-reducing drives only.

The tooth proportions that result from the data shown in this section or from the application of the original standards will be suitable for most speed-reducing (torque-increasing) applications. All gears, including pinions with small numbers of teeth, designed in accordance with the procedure shown will be free from undercut. In order to avoid the problems of undercut in pinions having fewer than the minimum standard number of teeth, each system shows proportions for long- and short-addendum teeth. Gears designed with long- and short-addendum teeth cannot be operated interchangeable on standard centers. In general, such gears should be designed to operate only as pairs. These gears can be cut with the same generating-type cutters and checked with the same equipment as standard-addendum gears. The proportions of long and short addendum shown in Table 4.1 are based on avoiding undercut. Such teeth will not have the optimum balance of strength and wear. Slightly different proportions can be used to achieve equal sliding balanced strength or a reduction in the tendency to score. Table 4.2 shows tooth proportions that have a good balance of strength and a minimum tendency to score.

**Standard tooth forms that have become obsolete.** Because industry design standards are continuously reviewed by the sponsoring organizations to ensure that the standards embody the most modern

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**TABLE 4.5**

**Basic Tooth Proportions of Spur Gears**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Item</th>
<th>Coarse Pitch (Coarser than 20P), Full Depth</th>
<th>Fine Pitch (20P and Finer), Full Depth</th>
<th>Explanation No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\phi)</td>
<td>Pressure angle</td>
<td>20°</td>
<td>25°</td>
<td>20°</td>
</tr>
<tr>
<td>(a)</td>
<td>Addendum (basic*)</td>
<td>(\frac{1.000}{P})</td>
<td>(\frac{1.000}{P})</td>
<td>(\frac{1.000}{P})</td>
</tr>
<tr>
<td>(b)</td>
<td>Dedendum (min.) (basic**)</td>
<td>(\frac{1.250}{P})</td>
<td>(\frac{1.250}{P})</td>
<td>(\frac{1.200}{P} + 0.002^*)</td>
</tr>
<tr>
<td>(h_k)</td>
<td>Working depth</td>
<td>(\frac{2.000}{P})</td>
<td>(\frac{2.000}{P})</td>
<td>(\frac{2.000}{P})</td>
</tr>
<tr>
<td>(h_t)</td>
<td>Whole depth (min.) (basic**)</td>
<td>(\frac{2.250}{P})</td>
<td>(\frac{2.250}{P})</td>
<td>(\frac{2.200}{P} + 0.002^*)</td>
</tr>
<tr>
<td>(t)</td>
<td>Circular tooth thickness (basic*)</td>
<td>(\frac{\pi}{2P})</td>
<td>(\frac{\pi}{2P})</td>
<td>(\frac{1.5708}{P})</td>
</tr>
<tr>
<td>(r_f)</td>
<td>Fillet radius (in basic rack)</td>
<td>(\frac{0.300}{P})</td>
<td>(\frac{0.300}{P})</td>
<td>Not standardized</td>
</tr>
<tr>
<td>(c)</td>
<td>Clearance (min.) (basic*)</td>
<td>(\frac{0.250}{P})</td>
<td>(\frac{0.250}{P})</td>
<td>(\frac{0.200}{P} + 0.002^*)</td>
</tr>
<tr>
<td>(c)</td>
<td>Clearance, shaved or ground teeth</td>
<td>(\frac{0.350}{P})</td>
<td>(\frac{0.350}{P})</td>
<td>(\frac{0.350}{P} + 0.002^*)</td>
</tr>
<tr>
<td>(N'P)</td>
<td>Min. numbers of teeth*:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pinion</td>
<td>18</td>
<td>12</td>
<td>18</td>
<td>10</td>
</tr>
<tr>
<td>Pair</td>
<td>36</td>
<td>24</td>
<td>–</td>
<td>11</td>
</tr>
<tr>
<td>(t_o)</td>
<td>Min. width of top land</td>
<td>(\frac{0.250}{P})</td>
<td>(\frac{0.250}{P})</td>
<td>Not standardized</td>
</tr>
</tbody>
</table>

* These values are basic, for equal-addendum gearing. When the gearing is made long- and short-addendum these values will be altered.

** These values are minimum. Shaved or ground teeth should be given proportions suitable to these processes.
### TABLE 4.6
Minimum Number of Pinion Teeth vs. Pressure Angle and Helix Angle Having No Undercut

<table>
<thead>
<tr>
<th>Helix Angle, Deg.</th>
<th>Minimum Number of Teeth to Avoid Undercut When Normal Pressure Angle, ϕ_{n}, Deg.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>14 ½</td>
</tr>
<tr>
<td>0° (spur gear)</td>
<td>32</td>
</tr>
<tr>
<td>5°</td>
<td>32</td>
</tr>
<tr>
<td>10°</td>
<td>31</td>
</tr>
<tr>
<td>15°</td>
<td>29</td>
</tr>
<tr>
<td>20°</td>
<td>27</td>
</tr>
<tr>
<td>23°</td>
<td>25</td>
</tr>
<tr>
<td>25°</td>
<td>24</td>
</tr>
<tr>
<td>30°</td>
<td>21</td>
</tr>
<tr>
<td>35°</td>
<td>18</td>
</tr>
<tr>
<td>40°</td>
<td>15</td>
</tr>
<tr>
<td>45°</td>
<td>12</td>
</tr>
</tbody>
</table>

*Note: Addendum 1/Pd whole depth 2.25/Pd.*

### TABLE 4.7
General Recommendations on Numbers of Pinion Teeth for Spur and Helical Power Gearing

(The maximum number of teeth in the range is based on providing a suitable balance between strength and wear capacity. The minimum number of teeth is intended to require either no enlargement to avoid undercutting or no more enlargement than necessary)

<table>
<thead>
<tr>
<th>Range of No. of Pinion Teeth</th>
<th>Ratio, m_G</th>
<th>Diametral Pitch, P_d</th>
<th>Hardness</th>
</tr>
</thead>
<tbody>
<tr>
<td>19 – 60</td>
<td>1 – 1.9</td>
<td>1 – 19.9</td>
<td>200 – 240 BHN</td>
</tr>
<tr>
<td>19 – 50</td>
<td>2 – 3.9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>19 – 45</td>
<td>4 – 8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>19 – 45</td>
<td>1 – 1.9</td>
<td>1 – 19.9</td>
<td>Rockwell C 33 – 38</td>
</tr>
<tr>
<td>19 – 38</td>
<td>2 – 3.9</td>
<td></td>
<td>Rockwell C 38 – 63</td>
</tr>
<tr>
<td>19 – 35</td>
<td>4 – 8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>19 – 30</td>
<td>1 – 1.9</td>
<td>1 – 19.9</td>
<td>Rockwell C 38 – 63</td>
</tr>
<tr>
<td>17 – 26</td>
<td>2 – 3.9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>15 – 24</td>
<td>4 – 8</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Notes: Small numbers of teeth in the pinion require special tooth proportions to avoid undercut. Gears containing prime numbers of teeth of 101 and over may be difficult to cut on the gear-manufacturing equipment available. In general, prime numbers over 101 and all numbers over 200 should be checked with the shop to be sure the teeth can be made.*
technology, there now exists a group of obsolete tooth form standards. On occasion, a designer is confronted with a situation in which a replacement gear must be made to mesh in a gear train conforming to one of these earlier standards.

Table 4.9 shows basic data for some of the obsolete standards. The use of this data for new designs is not recommended.

**Brown and Sharp system.** This system, see Table 4.9, was developed by the Brown and Sharp Company to replace the cycloidal tooth system. It was therefore given similar tooth proportions. It was intended to be cut by form-milling cutters. The departure from the true involute curve in this system is made to avoid the problems of undercut in pinions having small numbers of teeth. Backlash is achieved by feeding the cutter deeper than standard.

**AGMA 14.5° composite system.** This system, is interchangeable with the B and S system. Replacement parts may be designed from the data in Table 4.9.

**Fellows 20° stub tooth system.** In order to achieve a stronger tooth form for special drives, the Fellows Gear Shaper Company developed a stub tooth system in 1898. This system avoided the problem of tooth interference by the combined means of higher pressure angle and smaller values of addendum and dedendum. The pitch was specified as a combination of two standard diametral pitches; Thus, 10/12 (read ten-twelve). The circular pitch, pitch diameter, and tooth thickness are based on the first number, which is, 10, and are the same as for a standard 10-diametral-pitch gear. The addendum, dedendum, and clearance, however, are based on the second number (12) and are the same as for a 12-diametral-pitch gear.

**TABLE 4.8**

**Equations and References for Addendum Calculations, Spur Gears**

<table>
<thead>
<tr>
<th>Type of Tooth Design</th>
<th>Operating Condition</th>
<th>Pinion</th>
<th>Gear</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard or equal addendum</td>
<td>Numbers of teeth in gear and pinion greater than minimum numbers shown Tables 4.6 and 4.11</td>
<td>(a) ( \frac{1.00}{P_n} ); see Table 4.5</td>
<td>(a) ( \frac{1.00}{P_n} ); see Table 4.5</td>
</tr>
<tr>
<td>Long and short addendum</td>
<td>Numbers of teeth in pinion less than minimum shown in Table 4.6 and numbers of teeth in gear more than minimums shown in Table 4.11</td>
<td>(b) Value from Table 4.13</td>
<td>(b) Value from Table 4.13</td>
</tr>
<tr>
<td></td>
<td>Numbers of teeth in pinion less than minimum numbers shown in Table 4.11 and numbers of teeth in gear less than minimum numbers of teeth shown in Table 4.11</td>
<td>(c) ( \frac{P_1}{R_1} ) or</td>
<td>(c) ( \frac{P_1}{R_1} ) or</td>
</tr>
<tr>
<td></td>
<td>Gearing designed for balanced strength</td>
<td>(d) ( \frac{P_2}{R_2} ) if not less than (c)</td>
<td>(d) ( \frac{P_2}{R_2} )</td>
</tr>
<tr>
<td>Designed for nonstandard center-distance</td>
<td>Increase center-distance sufficiently to avoid undercut, in pinion and gear</td>
<td>(e) ( \frac{P}{R} )</td>
<td>See Section 4.3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(f) ( \frac{P_1}{R_1} ) or</td>
<td>Table 4.1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(g) Calculate tooth thickness to meet required center-distance from Eq. (4.75).</td>
<td>Table 4.2</td>
</tr>
</tbody>
</table>

*Notes:* The values in this table are for gears of 1 diametral pitch. For other sized divide by the required diametral pitch.

The values of addendum shown are the minimum increase necessary to avoid undercut. Additional addendum can be provided for special applications to balance strength. See Table 4.2.
AGMA 14.5° full-depth system. In this system, the tooth proportions are identical with those of the 14.5° composite system. The sides of the rack teeth are straight lines and therefore produce involute gear tooth profiles in the generating process. The minimum number of teeth to obtain full tooth action is 31 unless the teeth are modified.

Cycloidal tooth profiles. The cycloidal tooth profile is no longer used for any types of gears except clockwork and certain types of timer gears. A combination of involute and cycloidal tooth profile is found in the now-obsolete “composite system”. Clockwork gearing is based on the cycloid but has been greatly modified for practical reasons.

The cycloidal tooth is derived from the trace of a point on a circle (called the describing circle) rolling without slippage on the pitch circle of the proposed gear. The addendum portions of the tooth are the trace of a point on the describing circle rolling on the outside of the pitch circle. This trace is an epicycloid. The dedendum portion of the gear tooth is formed by a trace of a point on the describing circle rolling on the inside of the pitch circle. This trace is a hypocycloid.

Interchangeable systems of gears must have describing circles that are identical in diameter, and teeth that have the same circular pitch. The faces and flanks of the teeth must be generated by describing circles of the same size. The tooth proportions for cycloidal teeth were made to the same proportions as were adopted by the B and S system for gears.

### TABLE 4.9

Tooth Proportions of Spur Gears, Obsolete System*
(Tooth proportions for fully interchangeable gears operate on standard center distances)

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Item</th>
<th>Brown &amp; Sharp system</th>
<th>AGMA 14.5° composite system</th>
<th>Fellows 20° stub system</th>
<th>AGMA full-depth composite system</th>
</tr>
</thead>
<tbody>
<tr>
<td>ϕ</td>
<td>Pressure angle, deg.</td>
<td>14 ½</td>
<td>14 ½</td>
<td>20</td>
<td>14 ½</td>
</tr>
<tr>
<td>a</td>
<td>Addendum (basic**)</td>
<td>1.000</td>
<td>1.000</td>
<td>0.800</td>
<td>1.000</td>
</tr>
<tr>
<td>b</td>
<td>Dedendum (basic***)</td>
<td>1.157</td>
<td>1.157</td>
<td>1.000</td>
<td>1.157</td>
</tr>
<tr>
<td>h&lt;sub&gt;k&lt;/sub&gt;</td>
<td>Working depth</td>
<td>2.000</td>
<td>2.000</td>
<td>1.600</td>
<td>2.000</td>
</tr>
<tr>
<td>h&lt;sub&gt;1&lt;/sub&gt;</td>
<td>Whole depth (min.)</td>
<td>2.157</td>
<td>2.157</td>
<td>1.800</td>
<td>2.157</td>
</tr>
<tr>
<td>t&lt;sub&gt;k&lt;/sub&gt;</td>
<td>Circular tooth thickness (basic**)</td>
<td>$\frac{\pi}{2p}$</td>
<td>$\frac{\pi}{2p}$</td>
<td>$\frac{\pi}{2p}$</td>
<td>$\frac{\pi}{2p}$</td>
</tr>
<tr>
<td>r&lt;sub&gt;f&lt;/sub&gt;</td>
<td>Fillet radius (in basic rack)</td>
<td>0.157</td>
<td>0.157</td>
<td>Not standardized</td>
<td>1.33 × clearance</td>
</tr>
<tr>
<td>c&lt;sub&lt;k&lt;/sub&gt;</td>
<td>Clearance (min.)</td>
<td>0.157</td>
<td>0.157</td>
<td>0.200</td>
<td>0.157</td>
</tr>
<tr>
<td>c&lt;sub&gt;c&lt;/sub&gt;</td>
<td>Clearance, shaved or ground teeth</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N&lt;sub&gt;p&lt;/sub&gt;</td>
<td>Min. numbers of teeth**</td>
<td>32</td>
<td>–</td>
<td>14</td>
<td>32</td>
</tr>
<tr>
<td></td>
<td>Pinion</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Pair</td>
<td>64</td>
<td>–</td>
<td>–</td>
<td>64</td>
</tr>
</tbody>
</table>

* See Table 4.5.

** These values are basic for equal-addendum gearing. When the gearing is made long- and short-addendum these values will be altered.

*** These values are minimum. Shaved or ground teeth should be given proportions suitable to these processes.
The cycloidal system does not have a standard pressure angle. The operating pressure angle varies from zero at the pitch line to a maximum at the tips of the teeth.

In order to achieve correct meshing, the gears must be operated on centers that will maintain the theoretical pitch circles in exact contact. This was one of the major disadvantages of the cycloidal tooth form.

If the diameter of the describing circles were made equal to the radius of the pitch circle of the smallest pinion to be used in the system, the flanks of the teeth would be radial. This member, in effect, establishes the describing circle diameters of each system. In general, the systems for industrial gears were based on pinions of 12 to 15 teeth.

Clockwork and timer tooth profiles. The tooth form of most clockwork and timer gearing involving some type of escapement differs from other types of gearing because of the peculiar requirements imposed by the operating conditions. Two requirements of this type of gearing are outstanding:

- The gearing must be of the highest possible efficiency
- The gearing is speed increasing, that is, larger gears driving smaller pinions, with high ratios (between 6:1 and 12:1) due to the need for minimizing the number of gear wheels and economize on space.

As is discussed in Section 4.1.3 under “Low friction gearing”, the most efficient mode of tooth engagement is found in the arc of recess. In the case of speed-increasing involute profile drives, this means short-addendum pinions. This requirement may be in direct conflict with the need to make the pinion long addendum to avoid undercut. Clock gearing also does not have the requirement that it transmit smooth angular motion. A typical watch train will come to a complete stop five times a second. Furthermore, the motion and force are in one direction, minimizing the need for accurate control of backlash.

The modified cycloidal tooth form is the most commonly used form for the going train of clocks, timers, and watches. Experience has shown that the various modifications to this basic tooth have little effect on the performance because of the scale effect of tolerances required.

Specific spur gear calculation procedure. The following directions give a step-by-step procedure for determining spur gear proportions:

1. The application requirements should give the requirements of ratio, input speed, and kind of duty to be performed. The duty may be one of power transmission or of motion transmission
2. Based on application requirements, decide what pressure angle to use and plan to use a standard system (if possible) (see Table 4.5)
3. Pick approximate number of pinion teeth
   a. Consult Table 4.10 for general information
   b. Check Tables 4.6 and 4.11 for undercut conditions
   c. If power gearing, check Table 4.7 for data on balancing strength and wear capacity
4. Having approximate number of pinion teeth, determine approximate center distance and face width
5. Based on number of pinion teeth and center-distance, determine approximate pitch. Check Table 4.12 for standard pitches, and, if possible, use a standard pitch. Readjust pinion tooth numbers, center-distance, and ratio to agree with pitch chosen. See Chapter 2 for basic relations of pitch, center-distance, and ratio. Also use Eq. (4.8), Eq. (4.9), Eq. (4.10), and Eq. (4.11) if special “operating” center-distance is used
6. Determine whole depth of pinion and gear. Use Tables 4.5 and 4.13. Divide tabular value for 1 diametral pitch by actual diametral pitch to get design whole depth
7. Determine addendum of pinion and gear. Consult Tables 4.1, 4.2, 4.5, and 4.8. Follow these rules:
   a. Use enough long addendum to avoid undercut (see Table 4.1)
   b. If a critical power job—speed decreasing—balance addendum for strength (see Table 4.2)
   c. If no undercut or power problems, use standard addendum (see Table 4.5)

8. Determine operating circular pitch. If standard pitch is used, consult Table 4.14. If enlarged center-distance is used, determine operating circular pitch form operating pitch diameters. [Eq. (4.8) to Eq. (4.11)].

9. Determine design tooth thickness. Decide first on how much to thin teeth for backlash. Table 4.4 gives recommended amounts for power gearing having in mind normal accuracy of center-distance and normal operating temperature variations between gear wheels and casings. If special designs with essentially “no backlash” or unusual materials and accuracy are involved, consult the last part of this chapter, titled “Center-Distance”, for special calculations. If the pinion and gear have equal addendum, make the theoretical tooth thicknesses equal, and do this by dividing the circular pitch by 2. If a long-addendum pinion is used with a short-addendum gear, adjust tooth thicknesses by the following:

### TABLE 4.10
Numbers of Pinion Teeth, Spur Gearing

<table>
<thead>
<tr>
<th>No. of Teeth</th>
<th>Remarks</th>
</tr>
</thead>
</table>
| 7 – 9        | a. Requires long addendum to avoid undercut on all pressure angles  
               b. If 20˚, outside diameter should be reduced in proportion to tooth thickness to avoid pointed teeth  
               c. Should be made with 25˚ pressure angle if feasible  
               d. May result in poor contact ratio in very fine diametral pitches because of accumulation of tolerances  
               e. See Table 4.11 for minimum number of teeth in mating gear  
               f. Subject to high specific sliding and usually have poor wear characteristics |
| 10           | Smallest practical number with 20˚ pressure angle  
               a. Requires long addendum to avoid undercut if 20˚ pressure angle or less  
               b. Contact ratio may be critical in very fine pitches  
               c. See Table 4.11 for minimum number of teeth in mating gear |
| 12           | Smallest practical number for power gearing of pitches coarser than 16 diametral pitch  
               a. Requires long addendum to avoid undercut if 20˚ pressure angle or less  
               b. Smallest number of teeth that can be made “standard” if 25˚ pressure angle  
               c. About the minimum number of teeth for any good fractional-horsepower gear design where long life is important  
               d. See Table 4.11 for minimum number of teeth in mating gear |
| 15           | Used where strength is more important than wear  
               a. Requires long addendum to avoid undercut if 20˚ pressure angle or less  
               b. See Table 4.11 for minimum number of teeth in mating gear |
| 19           | Can be made standard addendum if 20˚ pressure angle or greater |
| 25           | Allows good balance between strength and wear for hard steels. Contact (contact diameter) is well away from critical base-circle region. |
| 35           | If made of hard steels, strength may be more critical than wear. If made of medium-hard (Rockwell C 30) steels strength and wear about equal |
| 50           | Excellent wear resistance. Favored for high-speed gearing because of quietness. Critical on strength on all but low-hardness pinions |
### TABLE 4.11
Numbers of Teeth in Pinion and Gear vs. Pressure Angle and Center-Distance

<table>
<thead>
<tr>
<th>No. of Teeth in Pinion</th>
<th>14 ½ Coarse Pitch*</th>
<th>20 Coarse Pitch**</th>
<th>20 Fine Pitch**</th>
<th>25 Coarse Pitch**</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>7 42***</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>8 39***</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>9 36***</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>10 35</td>
<td>25</td>
<td>33</td>
<td>15</td>
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<tr>
<td>11</td>
<td>11 34</td>
<td>24</td>
<td>30</td>
<td>14</td>
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<tr>
<td>12</td>
<td>12 33</td>
<td>23</td>
<td>27</td>
<td>12</td>
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<tr>
<td>13</td>
<td>13 32</td>
<td>22</td>
<td>25</td>
<td></td>
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<tr>
<td>14</td>
<td>14 31</td>
<td>21</td>
<td>23</td>
<td></td>
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<tr>
<td>15</td>
<td>15 30</td>
<td>20</td>
<td>21</td>
<td></td>
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<tr>
<td>16</td>
<td>16 29</td>
<td>19</td>
<td>19</td>
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<tr>
<td>17</td>
<td>17 28</td>
<td>18</td>
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<tr>
<td>18</td>
<td>18 27</td>
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<td></td>
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<tr>
<td>19</td>
<td>19 26</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>20</td>
<td>20 25</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>21</td>
<td>21 24</td>
<td></td>
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<tr>
<td>22</td>
<td>22 23</td>
<td></td>
<td></td>
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<td>23</td>
<td>23 22</td>
<td></td>
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<tr>
<td>24</td>
<td>24 21</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>25</td>
<td>25 20</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>26</td>
<td>26 19</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>27</td>
<td>27 18</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>28</td>
<td>28 17</td>
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<td>29</td>
<td>29 16</td>
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</tr>
<tr>
<td>30</td>
<td>30 15</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>31</td>
<td>31 14</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Note: Pinions having fewer than 10 teeth are not recommended. Gears having fewer teeth than shown for any given pinion-gear combination will require an enlarged center-distance for proper operation.

*Gears of this pressure angle not recommended for new designs.

**Gears with these numbers of teeth can be made standard addendum and operated on standard center-distances.

***Not recommended; but if essential, use these values.

### TABLE 4.12
Recommended Diametral Pitches

<table>
<thead>
<tr>
<th>Coarse Pitch</th>
<th>Fine Pitch</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>20</td>
</tr>
<tr>
<td>2.25</td>
<td>24</td>
</tr>
<tr>
<td>2.5</td>
<td>32</td>
</tr>
<tr>
<td>3</td>
<td>40</td>
</tr>
</tbody>
</table>

*Note: These diametral pitches are suggested as a means of reducing the great amount of gear-cutting tooling that would have to be inventoried if all possible diametral pitches were to be specified.*
If the pinion only is enlarged and the center-distance is enlarged to accommodate a standard gear, enlarge the pinion tooth thickness only (see Table 4.14).

After the theoretical tooth thicknesses are obtained, subtract one-half the amount the teeth are to be thinned for backlash from the pinion and gear theoretical tooth thicknesses. This is the maximum design tooth thickness. Obtain the minimum design tooth thickness by subtracting a reasonable tolerance for machining from the maximum design tooth thickness.

10. Recheck load capacity using design proportions just obtained. If not within allowable limits, change design.

11. If the gear design is for critical power gears, additional items will need calculation:
   a. Root fillet radius [(4.37)]
   b. Form diameter
   c. Modification of profile [(4.35), (4.36)]
   d. Diameter over pins (see corresponding section below)

12. Certain general dimensions must be calculated and tolerated:
   a. Outside diameter, \( D + 2 (a \pm \Delta a) \)
   b. Root diameter, \( D_h - 2 h_r \)
   c. Face width
   d. Chordal addendum and chordal thickness

13. It may be necessary to specify:
   a. Tip round (at outside diameter) (see Table 4.14)
   b. Edge round (see Table 4.14)
   c. Roll angle
   d. Base radius

14. In some unusual designs, it may be necessary to check the following to assure a sound design:
   - Diameter at which teeth become pointed [(4.41)]
   - Width of top land [(4.38)]
   - Effective contact ratio [Eq. (4.3) and Eq. (4.4)]
   - Undercut diameter

\[
\Delta t = \Delta a 2 \tan \phi_n \quad (4.42)
\]

*Does not apply to pinions containing nine teeth or less.
1. **Pressure angle.** The pressure angle and numbers of teeth in a given pair of gears are related in that low pressure angles (14 ½) should be avoided for low numbers of teeth (see Tables 4.6 and 4.11).

2. **Addendum.** For general applications, use equal-addendum teeth when the numbers of teeth in the pinion and in the pair exceed the values shown in items 10 and 11. Use long addendums for pinions having numbers of teeth, pressure angles, and diametral pitches as shown in Table 4.1. The amount of decrease of addendum in the mating gear is to be limited to values that will not produce undercut.

The values of long addendum for pinions are limited to speed-decreasing drives. Data on speed-increasing drives are given in Section 4.1.3.
3. **Dedendum.** The values shown in the table are the minimum standard values. Shaved or ground teeth should be given a greater dedendum (whole depth and clearance values); see items 5 and 9. A constant value, 0.002 in., is added to the dedendum of fine-pitch gears, which allows space for the accumulation of foreign matter at the bottoms of spaces. This provision is particularly important in the case of very fine diametral pitches.

4. **Working depth.** The working depth customarily determines the type of tooth; that is, a tooth with a 1.60/P working depth is called a **full-depth tooth**.

5. **Whole depth.** The value shown is the standard minimum whole depth. It will increase in proportion to the increase in backlash cut into the teeth (unless the outside diameter is also correspondingly adjusted). It will also increase slightly if long- and short-addendum teeth are generated with pinion-shaped cutters. See also items 3, 8, and 9 for increase due to manufacturing processing requirements.

6. **Circular tooth thickness, basic.** This is the basic circular tooth thickness on the standard pitch circle. These values will be slightly altered if backlash is introduced into the gears to allow them to mesh at a standard center-distance. These values will be drastically altered in the case of long- and short-addendum designs. Table 4.15 gives values of tooth thickness corresponding to standard values of long and short addendum for small numbers of teeth.

7. **Fillet radius, basic rack.** The fillet radius shown is that in the basic rack. The tooth form standard directs that the edge radius on hobs and rack-type shaper cutters should be equal to the fillet radius in the basic rack. It also directs that pinion-shaped cutters should be designed using the basic rack as a guide so that the gear teeth generated by these cutters will have a fillet radius approximating those produced by hobs and rack-type shaper cutters. It can be calculated approximately by the data shown in (4.42).

---

**TABLE 4.15**

<table>
<thead>
<tr>
<th>No. of Teeth in Pinion</th>
<th>Coarse-Pitch System</th>
<th>Fine Pitch System</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>20° Pressure Angle</td>
<td>25° Pressure Angle</td>
</tr>
<tr>
<td>Pinion</td>
<td>Gear</td>
<td>Pinion</td>
</tr>
<tr>
<td>7</td>
<td>1.9120</td>
<td>1.7420</td>
</tr>
<tr>
<td>8</td>
<td>1.8680</td>
<td>1.6590</td>
</tr>
<tr>
<td>9</td>
<td>1.8260</td>
<td>1.5708</td>
</tr>
<tr>
<td>10</td>
<td>1.7830</td>
<td>1.5708</td>
</tr>
<tr>
<td>11</td>
<td>1.7410</td>
<td>1.5708</td>
</tr>
<tr>
<td>12</td>
<td>1.7090</td>
<td>1.5708</td>
</tr>
<tr>
<td>13</td>
<td>1.6770</td>
<td>1.5708</td>
</tr>
<tr>
<td>14</td>
<td>1.6450</td>
<td>1.5708</td>
</tr>
<tr>
<td>15</td>
<td>1.6130</td>
<td>1.5708</td>
</tr>
<tr>
<td>16</td>
<td>1.5810</td>
<td>1.5708</td>
</tr>
<tr>
<td>17</td>
<td>1.5490</td>
<td>1.5708</td>
</tr>
<tr>
<td>18</td>
<td>1.5170</td>
<td>1.5708</td>
</tr>
<tr>
<td>19</td>
<td>1.4850</td>
<td>1.5708</td>
</tr>
</tbody>
</table>

*Note:* These tooth thicknesses go with Table 4.1 addendums.

The above values are for 1 diametral pitch.

These basic tooth thicknesses do not include an allowance for backlash.
In the case of 25°– pressure-angle teeth, the fillet radius shown must be reduced for teeth having a clearance of 0.250/P. This is discussed in Section 4.1.4.

In the case of fine-pitch teeth, the fillet radius usually will be larger than the clearance customarily given as \( c = 0.157/P \) because of edge breakdown in the cutter. The effects of tool wear on fine-pitch gears are proportionally larger than the effects produced by coarse-pitch tools.

8. **Clearance.** The value shown is the minimum standard. Greater clearance is usually required for teeth that are finished by grinding or shaving. In general, the value shown in item 9 will be suitable for these processes. See also items 3 and 5.

9. **Clearance for shaved or ground teeth.** This is the recommended clearance for teeth to be finished by shaving or grinding. In the case of 25°–pressure-angle teeth, the fillet radius shown must be reduced for teeth having a clearance of 0.250/P.

10. **Minimum number of teeth in pinion.** These are the lowest numbers of teeth that can be generated in pinions having standard addendums and tooth thicknesses that will not be undercut. Pinions having fewer teeth should be made long addendum in accordance with Table 4.1.

11. **Minimum numbers of teeth in pair.** This is the smallest number of teeth in pinion and gear that can be meshed on a standard center-distance without one member’s being undercut. For pairs with fewer teeth the members will have to be meshed on a nonstandard (enlarged) center-distance.

12. **Minimum width of top land.** This is the approximate minimum width of top land allowable in standard long-addendum pinions. Increases in addendum that cause the tops of the teeth to have less than this value should be generally avoided.

### 4.2.2 System for Helical Gears

The following data are based on the information contained in AGMA standard for “20-Degree Involute Fine-Pitch System, for Spur and Helical Gears”. In addition, tooth proportions are shown that are used by several of the larger gear manufacturers for the design of helical gears. These tooth proportions are shown since there are no AGMA standards for coarse-pitch helical gears.

In general, helical tooth proportions are based on either getting the most out of a helical gear design or on using existing tooling. The tooth proportions referred to above have been found to yield very good gears. In some cases, for less critical applications, tools on hand may be used. Very often hobs for spur gears are available, and gears for general service can be made with these tools. Since such hobs have no taper, they are not well suited to cut helix angles much over 30°.

In helical gear calculations, care should be taken to avoid confusion as to which plane the various tooth proportions are measured in. In some cases, the transverse plane is used. This plane is perpendicular to the axis of the helical gear blank. In some cases, it is desirable to work in the normal plane.

If a spur gear hob is used to cut helical teeth, the relationship between the transverse and normal pressure angles and the transverse and normal pitches as well as the base pitches should be established:

\[
P_d = \frac{\pi \cos \psi}{p_n} \quad (4.43)
\]

\[
\tan \phi = \frac{\tan \phi_n}{\cos \psi} \quad (4.44)
\]

\[
p_N = p_n \cos \phi_n \quad (4.45)
\]
where:

- \( P_d \) – is the diametral pitch (in transverse plane)
- \( \psi \) – is the helix angle
- \( p_n \) – is the circular pitch (in normal plane)
- \( \phi_n \) – is the pressure angle (in normal plane)
- \( p_n \) – is the normal base pitch (normal to surface)

When gears mesh together or with hobs, racks, shaper, or shaving cutters, their normal base pitches must be equal.

When a gear shaper is to be used to cut helical teeth, special guides are installed in the machine. These impart the twist into the cutter spindle, which produces the proper helix angle and lead. The cutter used has the same lead ground into the teeth as is produced by the guides. The cutter and guides in turn produce a given lead angle to the gear being cut. Practical considerations limit the lead angle on the guides to roughly 35°. In order to control costs, helical gears that are to be shaped should be designed either to a standard series of values of leads, or to values of lead available in existing guides.

Equation (4.46) shows the relationship of lead, number of teeth, diametral pitch, and helix angle in a given cutter, or helical gear:

\[
\tan \psi = \frac{\pi N}{P_d L}
\]  

(4.46)

where:

- \( \psi \) – is the helix angle
- \( N \) – is the number of teeth, cutter or gear
- \( P_d \) – is the diametral pitch
- \( L \) – is the lead of cutter

**Selection of tooth form.** The tooth forms shown in Table 4.3 are used for ranges of applications as follows (see Table 4.16). The foregoing types each require a separate set of tooling. They have the advantage of getting the most out of good helical gear design.

When an existing spur gear hob is to be used to produce the gear, use types 18 and 19 tooth forms. The data in Table 4.17 will be found helpful in making calculations for gears having types 18 and 19 tooth forms. Type 19 tooth form is similar to type 18 tooth form except that its proportions are based on the use of hobs designed to cut fine-pitch gears (20 diametral pitch or finer).

**Selection of helix angle.** Single helical gears usually are given lower helixes angles than double helical gears in order to limit thrust loads. Typical helix angles 15° and 23°. Low helix angles do not provide so many axial crossovers as can be achieved on high-helix-angle gears of a given face width. Double-helix gears have helix angles that typically range from 30° through 45°. Although higher helix angles provide smoother operation, the tooth strength is lower.

In order to get the quietest gears and at the same time achieve good tooth strength, special cutting tools for each helix angle should be provided. Table 4.3 shows typical tooth proportions. Tooth types 1 through 5 require special cutters, and the helix angle shown should be used. These teeth are stubbed more and more as the helix angle is higher. Tooth types 6 through 12 can be made with any helix angle desired. Types 13 and 14 can also be made with almost any helix angle. These tooth proportions are based on the use of standard spur gear hobs. Since such hobs are not usually tapered, they do not do as god a job is cutting as do the specially designed helical hobs, and, as a result, they should be limited in use to helix angles less than 25°.
Face width. In the design of helical gears, the face width is usually based on that needed to achieve the required load-carrying capacity. In addition, the face width and lead are interrelated in that it is necessary to obtain at least two axial pitches of face width \((F = 2p_x)\) to get reasonable benefit from the helical action, and four or more if high speeds, noise, or critical designs are confronted.

Long- and short-addendum designs are not as common among helical gears as among spur gears. This is because a much lower number of teeth can be cut in helical gears without undercut. In low-hardness helical gearing, pitting, which is little affected by changes in addendum, is usually the limiting feature. In high-hardness designs, the same proportions as those used for spur gear addendums should be employed.

Specific calculation procedure for helical gears. The procedure for helical gears is very similar to that given in Section 4.2.1 for spur gears:

1. Determine application requirements: ratio, power, speed, and so forth
2. Decide on basis tooth proportions and helix angle (see Table 4.3)
3. Pick appropriate number of pinion teeth (see Tables 4.6, 4.7, and 4.10)
4. Determine approximate center-distance and face width
5. Determine pitch of teeth
6. Determine whole depth
7. Determine addendum of pinion and gear. Use equal addendum for pinion and gear except in special cases where addendum must be increased to avoid undercut or special consideration must be given to increasing the pinion strength. (see Table 4.19)
8. Determine operating circular pitch, operating helix angle, and operating normal pitch if special center-distance was used
9. Determine design tooth thickness. If long and short addendum, adjust theoretical tooth thicknesses accordingly. Thin teeth for backlash. Consult Table 4.4 for power gearing backlash allowance
10. Recheck load capacity. Check number of axial crossovers [(4.5)]. Check ability of thrust bearings to handle thrust reactions. If results of these checks are unsatisfactory, change proportions
11. If gear design is for critical power gears, additional items may need calculation
   a. Root fillet radius [(4.37)]
   b. Form diameter

**TABLE 4.18**
Tooth Proportions for Helical Gears

<table>
<thead>
<tr>
<th>Helix Angle, Deg</th>
<th>Diametral Pitch, $P_d$</th>
<th>Circular Pitch, $P_t$</th>
<th>Axial Pitch, $p_x$</th>
<th>Pressure Angle*</th>
<th>Working Depth, $h_k$</th>
<th>Whole Depth, $h_t$ **</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.000000</td>
<td>3.14159</td>
<td>$\infty$</td>
<td>20°00'00&quot;</td>
<td>2.000</td>
<td>2.250</td>
</tr>
<tr>
<td>5</td>
<td>0.996195</td>
<td>3.15359</td>
<td>36.04560</td>
<td>20°4'13. 1&quot;</td>
<td>2.000</td>
<td>2.250</td>
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<tr>
<td>8</td>
<td>0.990268</td>
<td>3.17247</td>
<td>22.57327</td>
<td>20°10'50. 6&quot;</td>
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<td>2.250</td>
</tr>
<tr>
<td>10</td>
<td>0.984808</td>
<td>3.19006</td>
<td>18.09171</td>
<td>20°17'00. 7&quot;</td>
<td>2.000</td>
<td>2.250</td>
</tr>
<tr>
<td>12</td>
<td>0.978148</td>
<td>3.21178</td>
<td>15.11019</td>
<td>20°24'37. 1&quot;</td>
<td>2.000</td>
<td>2.250</td>
</tr>
<tr>
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<td>3.25242</td>
<td>12.13817</td>
<td>20°38'48. 8&quot;</td>
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</tr>
<tr>
<td>18</td>
<td>0.951057</td>
<td>3.30326</td>
<td>10.16640</td>
<td>20°56'30. 7&quot;</td>
<td>2.000</td>
<td>2.250</td>
</tr>
<tr>
<td>20</td>
<td>0.939693</td>
<td>3.34321</td>
<td>9.18540</td>
<td>21°10'22. 0&quot;</td>
<td>2.000</td>
<td>2.250</td>
</tr>
<tr>
<td>21</td>
<td>0.933580</td>
<td>3.36510</td>
<td>8.76638</td>
<td>21°17'56. 4&quot;</td>
<td>2.000</td>
<td>2.250</td>
</tr>
<tr>
<td>22</td>
<td>0.927184</td>
<td>3.38832</td>
<td>8.38636</td>
<td>21°25'57. 7&quot;</td>
<td>2.000</td>
<td>2.250</td>
</tr>
<tr>
<td>23</td>
<td>0.920505</td>
<td>3.41290</td>
<td>8.04029</td>
<td>21°34'26. 3&quot;</td>
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<td>2.250</td>
</tr>
<tr>
<td>24</td>
<td>0.913545</td>
<td>3.43890</td>
<td>7.72389</td>
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<td>2.250</td>
</tr>
<tr>
<td>25</td>
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<td>3.46636</td>
<td>7.43364</td>
<td>21°52'58. 7&quot;</td>
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<td>2.250</td>
</tr>
<tr>
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<td>3.49534</td>
<td>7.16651</td>
<td>22°02'44. 2&quot;</td>
<td>2.000</td>
<td>2.250</td>
</tr>
<tr>
<td>27</td>
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<td>3.52589</td>
<td>6.91994</td>
<td>22°13'10. 6&quot;</td>
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<td>2.250</td>
</tr>
<tr>
<td>28</td>
<td>0.882948</td>
<td>3.55807</td>
<td>6.69175</td>
<td>22°24'09. 0&quot;</td>
<td>2.000</td>
<td>2.250</td>
</tr>
<tr>
<td>29</td>
<td>0.874620</td>
<td>3.59195</td>
<td>6.48004</td>
<td>22°35'40. 0&quot;</td>
<td>2.000</td>
<td>2.250</td>
</tr>
<tr>
<td>30</td>
<td>0.866025</td>
<td>3.62760</td>
<td>6.28318</td>
<td>22°47'45. 1&quot;</td>
<td>2.000</td>
<td>2.250</td>
</tr>
</tbody>
</table>

*Pressure angle based on 20° normal pressure angle.

**The values shown for whole depth are for coarse-pitch gears. If the gears are to be shaved or ground, use $h_t = 2.35$. For fine-pitch gears, use $2.2/P_{ad} + 0.002$ for general purpose gearing and $2.35/P_{ad} + 0.002$ for gear to be shaved or ground.
TABLE 4.19
Equations and References for Addendum Calculations, Helical Gears

<table>
<thead>
<tr>
<th>Type of Tooth Design</th>
<th>Operating Conditions</th>
<th>Pinion</th>
<th>Gear</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard or equal addendum</td>
<td>Numbers of teeth in gear and pinion greater than minimum numbers shown in Figure 4.10</td>
<td>( a = \text{value from Table 4.3} )</td>
<td>If standard center-distance and values of ( a_p ) as shown at (a) left. If nonstandard center-distance, see item below</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( a = \frac{V_t}{P_n} ) ( a = \frac{V_t}{P_n} )</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>( a = \frac{V_t}{P_n} ) ( a = \frac{V_t}{P_n} )</td>
<td></td>
</tr>
<tr>
<td>Long- and short-addendum</td>
<td>Numbers of teeth in pinion less than minimum numbers shown in Table 4.6, and numbers of teeth in gear more than minimums shown in Figure 4.3</td>
<td>Increase addendum sufficiently to avoid undercut ( a_p = a + \frac{K'_n}{P_n} )</td>
<td>Decrease addendum by amount pinion addendum is increased. If undercut, increase center-distance. See item below</td>
</tr>
<tr>
<td></td>
<td></td>
<td>See Figure 4.10 for ( K'_n )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Numbers of teeth in pinion less than minimum numbers shown in Table 4.6, and numbers of teeth in gear less than minimum numbers of teeth shown in Figure 4.3</td>
<td>Increase addendum sufficiently to avoid undercut ( a_p = a + \frac{K'_n}{P_n} )</td>
<td>Increase addendum sufficiently to avoid undercut ( a_G = a + \frac{K'_n}{P_n} )</td>
</tr>
<tr>
<td>Designed for nonstandard center-distance</td>
<td>Increase center-distance by amount sufficient to accommodate increased addendum of both pinion and gear. Calculate tooth thickness to meet required center-distance from Eq. (4.75), then calculate required addendum from (4.18).</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
c. Modification of profile [(4.35), (4.36)]
d. Diameter over balls

12. General dimensions to be determined and toleranced
   a. Outside diameter
   b. Root diameter
   c. Face width
   d. Chordal addendum and chordal tooth thickness

13. It may be necessary to specify
   a. Tip round or chamber (see Table 4.16)
   b. Edge round or end bevel (see Table 4.16)
   c. Roll angle
   d. Base radius

14. Generally helical gears are not designed for such critical applications as to be in danger of pointed teeth or undercut.

4.2.3 SYSTEM FOR INTERNAL GEARS

In general, both spur internal meshes and helical internal meshes may be calculated by the same methods as external meshes. However, in internal gear meshes, there are several problems unique to this type of gearing.

The first of these is tip interference. In this type of interference, the pinion member cannot be assembled radially with the gear. Only axial assembly is possible, and it should be provided for in the design. If a shaper cutter having a number of teeth equal to or greater than the pinion is used to cut the internal gear, it will cut its way into mesh but in so doing will remove some material from the flanks of a few of the teeth that should have been left in place in order to have good tooth operation. This cutting action is also known as trimming. Such teeth will have poor contact and will tend to be noisy. If the proper shaper cutter or a broach is used, this problem will not occur.

The second problem is sometimes known as fouling. In this case, the internal gear teeth interfere with the flanks of the external tooth pinion if there is too small a difference in numbers of teeth between the pinion and the gear members.
Both these problems can be avoided in most gear designs by reducing the addendum of the internal gear (increasing its inside diameter). Tables 4.20 and 4.21 show a group of tooth proportions those will avoid these problems. A rather complicated graphical layout or involved calculations may be required to determine the exact proportions to avoid these problems. Since such calculations are beyond the scope of this chapter, Tables 4.20 and 4.21 are offered as a guide.

**Special calculations.** In general, the tip interference problem can be limited by providing more than a 17-tooth difference between gear and pinion.

The internal gear teeth have an addendum that extends toward the inside from the pitch circle. The critical dimension of addendum height is maintained by holding inside diameter.

### Table 4.20

<table>
<thead>
<tr>
<th>No. of Pinion Teeth</th>
<th>Pinion Addendum</th>
<th>Min. No. of Gear Teeth</th>
<th>Gear Addendum</th>
<th>Axial Assembly</th>
<th>Radial Assembly</th>
<th>Minimum Ratio</th>
<th>Ratio of 2</th>
<th>Ratio of 4</th>
<th>Ratio of 8</th>
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<tr>
<td>12</td>
<td>1.350</td>
<td>19 26</td>
<td>0.472</td>
<td>0.510</td>
<td>0.582</td>
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<td>0.729</td>
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<tr>
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<td>1.430</td>
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<td>0.574</td>
<td>0.642</td>
<td>0.733</td>
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<tr>
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<td>1.400</td>
<td>26 35</td>
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<td>0.734</td>
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<tr>
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<td>0.577</td>
<td>0.621</td>
<td>0.666</td>
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<tr>
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<td>0.802</td>
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<td>0.620</td>
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<tr>
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<td>0.620</td>
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<td>0.797</td>
<td>0.814</td>
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</tr>
</tbody>
</table>

* Minimum ratio of number of gear teeth divided by number of pinion teeth for axial assembly.
In general, the addenda of internal gears are made considerably shorter than those for equivalent external gears to avoid interference. The effect of the gear wrapping around the pinion tends to increase contact ratio, and also the chances for pinion-fillet interference.

Internal gear-sets may be designed so that the pinion can be introduced into mesh at assembly by a radial movement. For any given number of teeth in the pinion, there must be a number of teeth in the gear that is somewhat greater than required if axial assembly were to be allowed.

If a minimum difference in numbers of pinion and gear teeth is desired, the designer must design the gear casings and bearings so that the pinion can be introduced into mesh with the gear in an axial direction.

**TABLE 4.21**
Addendum Proportions and Limiting Numbers of Teeth for Internal Spur Gears of 25° Pressure Angle

<table>
<thead>
<tr>
<th>No. of Pinion Teeth</th>
<th>Pinion Addendum</th>
<th>Min. No. of Gear Teeth</th>
<th>Gear Addendum</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Axial Assembly</td>
<td>Radial Assembly</td>
</tr>
<tr>
<td>12</td>
<td>1.000</td>
<td>17</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td>1.220</td>
<td>17</td>
<td>21</td>
</tr>
<tr>
<td>13</td>
<td>1.000</td>
<td>18</td>
<td>22</td>
</tr>
<tr>
<td></td>
<td>1.200</td>
<td>18</td>
<td>22</td>
</tr>
<tr>
<td>14</td>
<td>1.000</td>
<td>19</td>
<td>24</td>
</tr>
<tr>
<td></td>
<td>1.180</td>
<td>19</td>
<td>24</td>
</tr>
<tr>
<td>15</td>
<td>1.000</td>
<td>20</td>
<td>25</td>
</tr>
<tr>
<td></td>
<td>1.170</td>
<td>20</td>
<td>25</td>
</tr>
<tr>
<td>16</td>
<td>1.000</td>
<td>21</td>
<td>26</td>
</tr>
<tr>
<td></td>
<td>1.150</td>
<td>21</td>
<td>26</td>
</tr>
<tr>
<td>17</td>
<td>1.000</td>
<td>22</td>
<td>27</td>
</tr>
<tr>
<td></td>
<td>1.140</td>
<td>22</td>
<td>27</td>
</tr>
<tr>
<td>18</td>
<td>1.000</td>
<td>23</td>
<td>28</td>
</tr>
<tr>
<td></td>
<td>1.130</td>
<td>23</td>
<td>28</td>
</tr>
<tr>
<td>19</td>
<td>1.000</td>
<td>24</td>
<td>29</td>
</tr>
<tr>
<td></td>
<td>1.120</td>
<td>24</td>
<td>29</td>
</tr>
<tr>
<td>20</td>
<td>1.000</td>
<td>25</td>
<td>30</td>
</tr>
<tr>
<td></td>
<td>1.110</td>
<td>26</td>
<td>29</td>
</tr>
<tr>
<td>22</td>
<td>1.000</td>
<td>28</td>
<td>32</td>
</tr>
<tr>
<td></td>
<td>1.090</td>
<td>28</td>
<td>32</td>
</tr>
<tr>
<td>24</td>
<td>1.000</td>
<td>30</td>
<td>34</td>
</tr>
<tr>
<td></td>
<td>1.080</td>
<td>30</td>
<td>34</td>
</tr>
<tr>
<td>26</td>
<td>1.000</td>
<td>32</td>
<td>37</td>
</tr>
<tr>
<td></td>
<td>1.060</td>
<td>32</td>
<td>37</td>
</tr>
<tr>
<td>30</td>
<td>1.000</td>
<td>36</td>
<td>40</td>
</tr>
<tr>
<td></td>
<td>1.050</td>
<td>36</td>
<td>40</td>
</tr>
<tr>
<td>40</td>
<td>1.000</td>
<td>46</td>
<td>51</td>
</tr>
<tr>
<td></td>
<td>1.020</td>
<td>46</td>
<td>51</td>
</tr>
</tbody>
</table>

* Minimum ratio of number of gear teeth divided by number of pinion teeth for axial assembly.
Tables 4.20 and 4.21 give the minimum numbers of teeth that a gear may have for any given pinion to achieve either radial or axial assembly.

The “hand” of helical internal gears is determined by the direction in which the teeth move, right or left, as the teeth recede from an observer looking along the gear axis. Whereas two external helical gears must be of opposite hand to mesh on parallel axes, an internal helical gear must be of the same hand as its mating pinion.

Specific calculation procedure for internal gears. Generally speaking, internal gears may be designed by the same procedure as outlined in Section 4.2.1 for spur gears, or Section 4.2.2 for helical gears. However, there are some special considerations:

1. Number of teeth. The number of teeth in the pinion is based on the ratio required and also on tooth strength and wear considerations. In addition, there must be sufficiently large difference between the numbers of teeth in the pinion and gear to avoid problems of tip interference. Tables 4.20 and 4.21 give the minimum numbers of teeth in gear and pinion that can be specified and still be able to achieve either axial or radial assembly.

2. Number of teeth, gear. The minimum number of teeth in the gear member that can be used without getting into problems of assembly techniques or tip interference problems is shown in Tables 4.20 and 4.21. The manufacturing organization that will produce the gears should be checked to determine the availability of suitable equipment when the number of teeth in the gear exceeds 200, or when prime numbers over 100 are used. If the internal gear is to be broached, consideration should be given to minimizing the number of teeth in the gear so as to keep broach cost down.

3. Helix angle. Since shaper guides are usually required for the cutting of the internal gear, first choice in helix angles should be based on existing guides. Because of the kinematics of the gear shaper, helix angle above about 30˚ should be avoided. The helix angle of the pinion member is of the same hand as that of the gear.

4. Diametral pitch. Since special tools are usually required to produce the internal gear, thought should be given to standardizing on ranges of numbers of teeth and diametral pitches for internal gearing. Table 4.12 shows suggested diametral pitches. Internal gearing can be made long and short addendum so that the need of nonstandard pitches to meet special center-distances is virtually nonexistent.

5. Normal circular pitch. In helical internal gears the normal circular pitch may be specified on the internal gear cutter in the case of existing cutters.

4.2.4 Standard Systems for Bevel Gears

It is well established practice to follow the latest standards on gearing. If manufacturing equipment built in German, Swiss, or Japanese companies is in use, appropriate standard data should be obtained from the manufacturer.

For the general guidance of the reader, the principal characteristics of systems now in use are discussed.

Discussion of 20˚ straight bevel gear system. The tooth form of the gears in this system is based on a symmetrical rack. In order to avoid undercut and to achieve approximately equal strength, a different value of addendum is employed for each ratio. If these gears are cut on modern bevel gear generators, they will have a localized tooth bearing. The selection of addendum ratios and outside diameter is limited to a 1:1.5 ratio in width of top lands of pinion and gear. The face cone of the gear and pinion blanks is made parallel to the root cone elements to provide parallel clearance. This permits the use of larger edge radii on the generating tools with the attendant greater fatigue strength. Figure 4.11 shows data on the relation of dedendum angle to undercut for straight bevel gears.

Discussion of spiral bevel gear system. Tooth thicknesses are proportioned so that the stresses in the gear and pinion will be approximately equal with the left-hand pinion driving clockwise or a left-hand
pinion driving counterclockwise. The values shown apply for gears operated below their endurance limits. For gears operated above their endurance limits, special proportions will be required. Special proportions will also be required for reversible drives on which optimum load capacity is desired. The method of establishing these balances is beyond the scope of this chapter but may be found in the *Gleason Works* publication “Strength of Bevel and Hypoid Gears”.

The tooth proportions are based on the 35° spiral angle. A smaller angle may result in undercut and a reduction in contact ratio (see Figure 4.12).

**Discussion of Zerol bevel gear system.** The considerations of tooth proportions to avoid undercut and loss of contact ratio as well as achieve optimum balance of strength are similar in this system to those of the straight and spiral systems.

This system is based on the duplex cutting method in which the root cone elements do not pass through the pitch cone apex. The face cone of the mating member is made parallel to the root cone to produce a uniform clearance.

**Special tooth forms.** The teeth of bevel gears can be given special form to reduce manufacturing costs or for engineering reasons.

---

![Diagram](image-url)

**FIGURE 4.11** Relation between the Dedendum Angle and the Pitch Angle at Which Undercut Begins to Occur in Generating Straight Bevel Gears Using Sharp-Cornered Tools.
Because of the flexibility of the machining system used to cut most bevel gears, many forms are possible.

The *Formate*\textsuperscript{12} tooth form is often used for gearing of high ratios because of its manufacturing economies. The teeth of the gear member are cut without generation, thus saving time, and the extra generation required to produce a conjugate pair is taken on the pinion. Since there are fewer pinion teeth, less time is spent in generation than if both pinion and gear teeth were generated.

Another tooth form suitable for high-speed manufacture is the *Revacile*\textsuperscript{13} straight bevel gear. This special tooth form is generated by a large disc cutter, one space being completed with each cutter revolution.

Both these forms are beyond the scope of this chapter.

*Limitations in 20° straight bevel gear system.* The data contained in Table 4.22 apply only to gears that meet the following requirements:

- The standard pressure angle is 20°; in certain cases, depending on numbers of teeth, other pressure angles may be used (see Table 4.23).

\textsuperscript{12} Registered trademark of the *Gleason Works*, Rochester, NY.

\textsuperscript{13} Registered trademark of the *Gleason Works*, Rochester, NY.
### TABLE 4.22
Tooth Proportions of Standard Bevel Gears

<table>
<thead>
<tr>
<th>Item</th>
<th>Straight Teeth</th>
<th>Spiral Teeth</th>
<th>Zerol Teeth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure angle, deg.</td>
<td>20° standard</td>
<td>20° standard</td>
<td>20° basic, 22 ½ or 25° where needed</td>
</tr>
<tr>
<td>Working depth</td>
<td>2.000/P&lt;sub&gt;d&lt;/sub&gt;</td>
<td>1.700/P&lt;sub&gt;d&lt;/sub&gt;</td>
<td>2.000/P&lt;sub&gt;d&lt;/sub&gt;</td>
</tr>
<tr>
<td>Clearance</td>
<td>0.188/P&lt;sub&gt;d&lt;/sub&gt; + 0.002”</td>
<td>0.188/P&lt;sub&gt;d&lt;/sub&gt;</td>
<td>0.188/P&lt;sub&gt;d&lt;/sub&gt; + 0.002”</td>
</tr>
<tr>
<td>Face width</td>
<td>F ≤ A&lt;sub&gt;α&lt;/sub&gt;/3 or F ≤ 10/P&lt;sub&gt;d&lt;/sub&gt;</td>
<td>F ≤ A&lt;sub&gt;α&lt;/sub&gt;/3 or F ≤ 10/P&lt;sub&gt;d&lt;/sub&gt;</td>
<td>F ≤ 0.25A&lt;sub&gt;α&lt;/sub&gt; or F ≤ 10/P&lt;sub&gt;d&lt;/sub&gt;</td>
</tr>
<tr>
<td></td>
<td>do not exceed smaller value&lt;sup&gt;b&lt;/sup&gt;</td>
<td>whichever is smaller&lt;sup&gt;b&lt;/sup&gt;</td>
<td>whichever is smaller&lt;sup&gt;b&lt;/sup&gt;</td>
</tr>
<tr>
<td>Spiral angle</td>
<td>–</td>
<td>35°&lt;sup&gt;e&lt;/sup&gt;</td>
<td>0°</td>
</tr>
<tr>
<td>Min. No. of teeth in system</td>
<td>see Table 3.22</td>
<td>12</td>
<td>13</td>
</tr>
<tr>
<td>Whole depth</td>
<td>2.18/P&lt;sub&gt;d&lt;/sub&gt; + 0.002”</td>
<td>2.18/P&lt;sub&gt;d&lt;/sub&gt;</td>
<td>2.18/P&lt;sub&gt;d&lt;/sub&gt; + 0.002”</td>
</tr>
<tr>
<td>Diametral range</td>
<td>–</td>
<td>12 and coarser</td>
<td>3 and finer</td>
</tr>
<tr>
<td>AGMA ref.</td>
<td>208.02</td>
<td>209.02</td>
<td>202.02</td>
</tr>
</tbody>
</table>

**Notes**

a  20° is standard pressure angle for straight-tooth bevel gears. Table 4.23 shows ratios that may be cut with 14 ½ degree pressure angle teeth.

b  It the face width exceeds one-third the outer cone distance, the tooth is in danger of breakage in the event that tooth contact shifts to the small end of the tooth.

c  This is the minimum number of teeth in the basic system (see Table 4.23 for equivalent minimum number of teeth in the gear member).

d  20° is standard pressure angle for spiral bevel gears. Table 4.23 shows ratios that may be cut with a 16° degree pressure angle teeth.

e  35° is standard pressure angle. If smaller spiral angles are used, undercut may occur and the contact ratio may be less.

f  For gears of 10 diametral pitch and coarser, the teeth are often rough-cut 0.005 deeper to avoid having the finish blades cut on their ends.

g  On duplex Zero bevel gears 1 in. is the maximum face width in all cases.

### TABLE 4.23
Pressure Angle and Ratio, Minimum Number of Teeth in Gear and Pinion That Can Be Used with Any Given Pressure Angle

<table>
<thead>
<tr>
<th>Pressure Angle, Deg</th>
<th>Straight Tooth</th>
<th>Spiral Tooth</th>
<th>Zerol Tooth</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Pinion</td>
<td>Gear</td>
<td>Pinion</td>
</tr>
<tr>
<td>20 (standard)</td>
<td>16</td>
<td>16</td>
<td>17</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>17</td>
<td>16</td>
</tr>
<tr>
<td></td>
<td>14</td>
<td>20</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td>13</td>
<td>30</td>
<td>14</td>
</tr>
</tbody>
</table>

(Continued)
TABLE 4.23 (Continued)
Pressure Angle and Ratio, Minimum Number of Teeth in Gear and Pinion That Can Be Used with Any Given Pressure Angle

<table>
<thead>
<tr>
<th>Pressure Angle, Deg</th>
<th>Type of Bevel Gear</th>
<th>Gear Tooth Design</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Straight Tooth</td>
<td>Spiral Tooth</td>
</tr>
<tr>
<td></td>
<td>Pinion</td>
<td>Gear</td>
</tr>
<tr>
<td>14½</td>
<td>13</td>
<td>22</td>
</tr>
<tr>
<td></td>
<td>12</td>
<td>26</td>
</tr>
<tr>
<td>16</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>–</td>
<td>–</td>
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<tr>
<td></td>
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<td>–</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>22½</td>
<td>13</td>
<td>13</td>
</tr>
<tr>
<td></td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>–</td>
<td>–</td>
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<td>–</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>25</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>–</td>
<td>–</td>
</tr>
</tbody>
</table>
In all cases, full-depth teeth are used. Stub teeth are avoided because of the reduction in contact ratio, which may increase noise, and the reduction in wear resistance.

- Long- and short-addendum teeth are used throughout the system (except for 1:1 ratios) to avoid undercut and to increase the strength of the pinion.
- The face width is limited to between one-fourth and one-third cone distance. The use of greater face widths results in an excessively small tooth size at the inner ends of the teeth.

**Limitations in spiral bevel gear system.** This system is more limited in its application than the straight tooth system. The data in this system do not apply to the following:

- Automotive rear-axle drives
- Formate pairs
- Gears and pinions of 12 diametral pitch and finer, which are usually cut with one of the duplex spread-blade methods
- Gear cut spread-blade and pinion cut single-side, with a spiral angle of less than 20°
- Ratios with fewer teeth than those listed in Table 4.23
- Larger spiral bevel gears cut on the planning-type generators where the spiral angles should not exceed 30°

**Limitations in Zerol bevel gear system.** The data contained in Table 4.22 apply only to Zerol bevel gears that meet the following requirements:

- The standard pressure angle (basic) is 20°. Where needed to avoid undercut, 22 ½ and 25° pressure angles are standard (see Table 4.23).
- The face width is limited to 25% of the cone distance since the small-end tooth depth decreases even more rapidly as the face width increases because of the duplex taper. In duplex Zerol gears, 1 in. face is the maximum value in any case.

**General comments.** Table 4.7 will be found helpful in selecting the proper numbers of teeth for most power gearing applications. Table 4.23 gives the minimum numbers of teeth in the gear for each number of pinion teeth and pressure angle. In general, the more teeth that are in the pinion, the more quietly it will run and the greater will be its resistance to wear. The equipment used to produce bevel gearing imposes upper limits on numbers of teeth. In general, if the gear is to contain over 120 teeth if an even number or above 97 if a prime number, the manufacturing organization that is to produce the gears should be checked for capacity.

Table 4.23 gives the minimum numbers of teeth in the pinion for each number of teeth in the gear for each pressure angle. The minimum number of pinion teeth is based on consideration of undercut.

Table 4.24 shows addendum values that have been used in the past. These values are at the large end of the tooth (outer cone distance). These values show what was usually used in older designs that are in production. New standards that are now being developed may not agree with these values. (If a new standard is used, all values should be taken from the new standard—including addendum and backlash).

Table 4.25 shows typical backlash values for bevel gears. In the field of bevel gears, mounting distance settings can change backlash and change the tooth contact pattern. Usually, the desired contact is achieved when a bevel gear set is mounted at the specified mounting distances. Figure 4.13 shows a typical mounting surface and body dimensions for a bevel gear.

### 4.2.5 Standard Systems for Worm Gears

For general reference, Table 4.26 presents the tooth proportions for single-enveloping worm gears and for double-enveloping worm gears. These proportions are typical of past design practice.
The following rules apply to conventional single-enveloping worm gears:

- **General practice.** The following rules apply to conventional single-enveloping worm gears:
  - The worm axis is at right angle to the worm gear axis (90°)
  - The worm gear is hobbed. Except of a small amount of oversize, the worm gear hob has the same number of threads, the same tooth profile, and the same lead as that of the mating worm. (A slight change in lead may be made to compensate for oversize effects)

### TABLE 4.24

Bevel Gear Addendum (for 1 Diametral Pitch)

<table>
<thead>
<tr>
<th>Ratio ($m_c$)</th>
<th>Addendum</th>
<th>Ratio ($m_c$)</th>
<th>Addendum</th>
</tr>
</thead>
<tbody>
<tr>
<td>From</td>
<td>To</td>
<td>Straight and Zerol</td>
<td>Spiral</td>
</tr>
<tr>
<td>1.00</td>
<td>1.00</td>
<td>1.000</td>
<td>0.850</td>
</tr>
<tr>
<td>1.01</td>
<td>1.02</td>
<td>0.990</td>
<td>0.840</td>
</tr>
<tr>
<td>1.02</td>
<td>1.03</td>
<td>0.980</td>
<td>0.830</td>
</tr>
<tr>
<td>1.03</td>
<td>1.04</td>
<td>0.970</td>
<td>0.820</td>
</tr>
<tr>
<td>1.04</td>
<td>1.05</td>
<td>0.960</td>
<td>0.820</td>
</tr>
<tr>
<td>1.05</td>
<td>1.06</td>
<td>0.950</td>
<td>0.810</td>
</tr>
<tr>
<td>1.06</td>
<td>1.08</td>
<td>0.940</td>
<td>0.800</td>
</tr>
<tr>
<td>1.08</td>
<td>1.10</td>
<td>0.930</td>
<td>0.790</td>
</tr>
<tr>
<td>1.10</td>
<td>1.11</td>
<td>0.920</td>
<td>0.780</td>
</tr>
<tr>
<td>1.11</td>
<td>1.12</td>
<td>0.910</td>
<td>0.770</td>
</tr>
<tr>
<td>1.12</td>
<td>1.13</td>
<td>0.900</td>
<td>0.770</td>
</tr>
<tr>
<td>1.13</td>
<td>1.14</td>
<td>0.900</td>
<td>0.760</td>
</tr>
<tr>
<td>1.14</td>
<td>1.15</td>
<td>0.890</td>
<td>0.760</td>
</tr>
<tr>
<td>1.15</td>
<td>1.17</td>
<td>0.880</td>
<td>0.750</td>
</tr>
<tr>
<td>1.17</td>
<td>1.19</td>
<td>0.870</td>
<td>0.740</td>
</tr>
<tr>
<td>1.19</td>
<td>1.21</td>
<td>0.860</td>
<td>0.730</td>
</tr>
<tr>
<td>1.21</td>
<td>1.23</td>
<td>0.850</td>
<td>0.720</td>
</tr>
<tr>
<td>1.23</td>
<td>1.25</td>
<td>0.840</td>
<td>0.710</td>
</tr>
<tr>
<td>1.25</td>
<td>1.26</td>
<td>0.830</td>
<td>0.710</td>
</tr>
<tr>
<td>1.26</td>
<td>1.27</td>
<td>0.830</td>
<td>0.700</td>
</tr>
<tr>
<td>1.27</td>
<td>1.28</td>
<td>0.820</td>
<td>0.700</td>
</tr>
<tr>
<td>1.28</td>
<td>1.29</td>
<td>0.820</td>
<td>0.690</td>
</tr>
<tr>
<td>1.29</td>
<td>1.31</td>
<td>0.810</td>
<td>0.690</td>
</tr>
<tr>
<td>1.31</td>
<td>1.33</td>
<td>0.800</td>
<td>0.680</td>
</tr>
<tr>
<td>1.33</td>
<td>1.34</td>
<td>0.790</td>
<td>0.680</td>
</tr>
<tr>
<td>1.34</td>
<td>1.36</td>
<td>0.790</td>
<td>0.670</td>
</tr>
<tr>
<td>1.36</td>
<td>1.37</td>
<td>0.780</td>
<td>0.670</td>
</tr>
<tr>
<td>1.37</td>
<td>1.39</td>
<td>0.780</td>
<td>0.660</td>
</tr>
<tr>
<td>1.39</td>
<td>1.41</td>
<td>0.770</td>
<td>0.660</td>
</tr>
<tr>
<td>1.41</td>
<td>1.42</td>
<td>0.770</td>
<td>0.650</td>
</tr>
<tr>
<td>1.42</td>
<td>1.44</td>
<td>0.760</td>
<td>0.650</td>
</tr>
<tr>
<td>1.44</td>
<td>1.45</td>
<td>0.760</td>
<td>0.640</td>
</tr>
<tr>
<td>1.45</td>
<td>1.48</td>
<td>0.750</td>
<td>0.640</td>
</tr>
<tr>
<td>1.48</td>
<td>1.52</td>
<td>0.740</td>
<td>0.630</td>
</tr>
</tbody>
</table>

*Note: In the earlier standards and handbooks, the equations for addendum for all types of bevel gears wear $a_{wG} = \text{Table 4.24}/P$. The values used in this table should be used only when checking calculations based on earlier standards.*
Fine-pitch worms are usually milled or ground with a double-conical cutter or grinding wheel with an inclined angle of $40^\circ$ (tool pressure angle is $20^\circ$).

Coarser pitch worms are often made with a straight-sided milling or grinding tool—in American practice. Some European countries favor making the worm member in involute helicoid. Functionally, either type will work well in practice providing that the accuracy and fit of mating parts is of equal quality.

For double-enveloping worm gears, the defined profile is that of the worm gear. The worm is made with a special tool that has a shape similar to that of the worm gear. For the double-enveloping worm gear, the usual shape is straight-sided in the axial section and tangent to defined basic circle.

**Basic tooth forms for worm gearing.** The tooth forms of worm gearing have not been standardized to the same degree as tooth forms for spur gearing, for example. Since a special hob has to be made to cut the gears that will mesh with each design of worm, and since the elements that constitute good worm gearing design are less well understood, there has been little incentive to standardize.

The most general practice in worm gear design and manufacture is to establish the shape of the worm thread and then to design a hob that will generate teeth on the gear that are conjugate to those on the
This practice is not followed in the case of double-enveloping gearing since special tooling based on the shape of the gear member is required to generate the worm.

The shape of the teeth of the worm member is dependent on the size of the tool and the method used to cut the threads. These threads may be cut with a straight-sided V-shaped tool in a lathe or milled with double conical milling cutters of up to 6-in. diameter in a thread mill or hobbed with a special hob, or ground in a thread-grinding machine with a grinding wheel having a diameter up to 20 in. In each case the worm thread profile will be noticeably different, especially in the case of the higher lead angles. Worms can also be rolled, in which case the thread profile will be still different. It is essential, therefore, for the designer of worm gearing either to specify the manufacturing method to be used to make the worm or to specify the coordinates of the worm profile.

Specific calculations for worm gears. In designing worm gear units and making calculations several considerations should be kept in mind:

- The ratio is the number of worm gear teeth divided by the number of worm threads. (The pitch diameter of the gear divided by the pitch diameter of the worm is almost never equal to the ratio!)
- Due to the tendency of the worm gear to wear-in to best fit the worm, common factors between the number of worm threads and the number of worm gear teeth should be avoided
- The normal circular pitch of the worm and the worm gear must be the same. Likewise, the normal pressure angle of worm and the normal pressure angle of the worm gear must be the same
- The axial pitch of the worm and transverse circular pitch of the worm gear must be the same. In a like manner, the axial pressure angle of the worm must be the same as the transverse pressure angle of the worm gear
- The lead of the worm equals the worm axial pitch multiplied by the number of threads. The lead can be thought of as the axial advance of a worm thread in one turn (360°) of the worm

<table>
<thead>
<tr>
<th>Application</th>
<th>No. of Worm Threads</th>
<th>Cutter Pressure Angle</th>
<th>Addendum</th>
<th>Working Depth</th>
<th>Whole Depth</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Tooth Proportions for Single-Enveloping Worms and Worm Gears</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Index or holding mechanism</td>
<td>1 or 2</td>
<td>14.5°</td>
<td>1.000</td>
<td>2.000</td>
<td>2.250</td>
</tr>
<tr>
<td>Power gearing</td>
<td>1 or 2</td>
<td>20°</td>
<td>1.000</td>
<td>2.000</td>
<td>2.250</td>
</tr>
<tr>
<td></td>
<td>3 or more</td>
<td>25°</td>
<td>0.900</td>
<td>1.800</td>
<td>2.050</td>
</tr>
<tr>
<td>Fine pitch (instrument)</td>
<td>1 – 10</td>
<td>20°</td>
<td>1.000</td>
<td>2.000</td>
<td>2.200+0.002 in. English</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>2.200+0.002 mm Metric</td>
</tr>
<tr>
<td><strong>Tooth Proportions for Double-Enveloping Worms and Worm Gears</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Item</td>
<td>No. of worm threads</td>
<td>Pressure angle</td>
<td>Addendum</td>
<td>Working depth</td>
<td>Whole depth</td>
</tr>
<tr>
<td>Power gearing</td>
<td>1 – 10</td>
<td>20°</td>
<td>0.700</td>
<td>1.400</td>
<td>1.600</td>
</tr>
</tbody>
</table>

Notes: The addendum, working-depth, and whole-depth values are for 1 normal diametral pitch. (The normal diametral pitch of the worm is equal to 3.141593 divided by the axial pitch of the worm and then the result divided by cosine of the worm lead angle).

In the English system, for normal diametral pitches other than 1, divided by the normal diametral pitch. In the metric system the values are for 1 normal module. For modules other than 1, multiply by the normal module.
• It has been customary to thin the worm threads to provide backlash but not thin the worm gear teeth
• There is a small difference between the pressure angle of a straight-sided tool used to mill or grind worm threads and the normal pressure angle of the worm thread

Table 4.26 shows typical tooth proportions for single-enveloping worm gear sets and for double-enveloping worm gear-sets. This table shows the addendum is one-half the working depth. In some cases, it may be described to make the worm addendum larger than the addendum of the worm gear.

Table 4.27 shows recommended minimum numbers of teeth for single-enveloping worm gears. Since the pressure angle changes going across the face width of the worm gear, larger numbers of teeth are needed to avoid undercut problems in worm gears than in spur gears.

Table 4.28 shows that lead angle needs to increase with the numbers of threads. If, for instance, a lead angle of 20° is desired to get good efficiency, the designer should plan to use 4 or 5 threads on the worm.

Worm gears are normally sized by picking the number of threads and teeth and then choosing a normal circular pitch that will give enough center-distance to have the necessary load-carrying ability. Table 4.29 shows suggested normal circular pitches.

Table 4.30 shows nominal backlash values for worm gear sets. These values represent thinning of worm threads and tolerances on worm thread thickness and gear tooth thickness.

Table 4.31 shows recommended cutter or grinding wheel outside diameters for the making of single-enveloping worms.

Further details in worm gear design and rating are given below in consequent chapters of the book.

---

**TABLE 4.27**

<table>
<thead>
<tr>
<th>Pressure Angle, Deg</th>
<th>Min. No. of Teeth</th>
</tr>
</thead>
<tbody>
<tr>
<td>14½</td>
<td>40</td>
</tr>
<tr>
<td>20</td>
<td>25</td>
</tr>
<tr>
<td>25</td>
<td>20</td>
</tr>
<tr>
<td>30</td>
<td>15</td>
</tr>
</tbody>
</table>

---

**TABLE 4.28**

<table>
<thead>
<tr>
<th>No. Worm Threads</th>
<th>Lead Angle, Deg</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Less than 6</td>
</tr>
<tr>
<td>2</td>
<td>3 to 12</td>
</tr>
<tr>
<td>3</td>
<td>6 to 18</td>
</tr>
<tr>
<td>4</td>
<td>12 to 24</td>
</tr>
<tr>
<td>5</td>
<td>15 to 30</td>
</tr>
<tr>
<td>6</td>
<td>18 to 36</td>
</tr>
<tr>
<td>7 and higher</td>
<td>(not over 6° per thread)</td>
</tr>
</tbody>
</table>
## TABLE 4.29
Some Recommended Normal Circular Pitches

<table>
<thead>
<tr>
<th>Fine Pitch</th>
<th>Coarse Pitch</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>English, in.</strong></td>
<td><strong>Metric, mm</strong></td>
</tr>
<tr>
<td>0.030</td>
<td>1.000</td>
</tr>
<tr>
<td>0.040</td>
<td>1.250</td>
</tr>
<tr>
<td>0.050</td>
<td>1.500</td>
</tr>
<tr>
<td>0.065</td>
<td>2.000</td>
</tr>
<tr>
<td>0.080</td>
<td>2.500</td>
</tr>
<tr>
<td>0.100</td>
<td>3.000</td>
</tr>
<tr>
<td>0.130</td>
<td>3.500</td>
</tr>
<tr>
<td>0.160</td>
<td>4.000</td>
</tr>
<tr>
<td>1.250</td>
<td></td>
</tr>
<tr>
<td>1.500</td>
<td></td>
</tr>
<tr>
<td>2.000</td>
<td></td>
</tr>
</tbody>
</table>

*Note: The normal circular pitch of the worm gear is equal to the axial pitch of the worm multiplied by the cosine of the lead angle.*

## TABLE 4.30
Recommended Values of Backlash for Single-Enveloping Worm Gearing and Double-Enveloping Worm Gearing

<table>
<thead>
<tr>
<th>Center-Distance</th>
<th>Backlash (amount worm should be reduced)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.003 0.008</td>
</tr>
<tr>
<td>6</td>
<td>0.006 0.012</td>
</tr>
<tr>
<td>12</td>
<td>0.012 0.020</td>
</tr>
<tr>
<td>24</td>
<td>0.018 0.030</td>
</tr>
</tbody>
</table>

## TABLE 4.31
Recommended Values of Cutter Diameter (Single-Enveloping Gear Sets)

<table>
<thead>
<tr>
<th>Type of Worm</th>
<th>Suggested Cutter Diameter, Dc, in.</th>
<th>Process</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low-speed power</td>
<td>4</td>
<td>Milled</td>
</tr>
<tr>
<td>High-speed power</td>
<td>20</td>
<td>Ground</td>
</tr>
<tr>
<td>Fine-pitch:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Commercial quality</td>
<td>3</td>
<td>Milled</td>
</tr>
<tr>
<td>Precision quality</td>
<td>20</td>
<td>Ground</td>
</tr>
</tbody>
</table>
4.2.6 **STANDARD SYSTEM FOR FACE GEARS**

Fine pitch (20 diametral pitch and finer) face gears have been standardized. Coarse pitch face gears have not been standardized. The following data are arranged to provide a logical method of calculating the tooth proportions of face gears. Such gears will be suitable for most applications. Table 4.32 shows basic tooth proportions for face gear sets.

Caution should be exercised in using Table 4.32. The items shown apply only to gears that meet the following requirements:

- The axes of the gear and the pinion must intersect at an angle of $90^\circ$
- The gear must be generated by means of a reciprocating pinion-shaped cutter having the same diametral pitch and pressure angle as the mating pinion and must be of substantially the same size
- The pinion should be sized to meet requirements on load-carrying capacity
- The minimum number of teeth in the pinion in this system is 12, and the minimum cutter pitch diameter is 0.250 in.
- The minimum gear ratio is 1.5:1 and the maximum ratio is 12.5:1
- The long- and short-addendum designs for the pinion with less than 18 teeth should not generally be used on speed-increasing drives

**Pinion design.** The pinion member in case of numbers of teeth less than 18 has an enlarged tooth thickness to avoid undercut when cut with a standard cutter (Tables 4.33 and 4.34). Compared with the pinions designed in accordance with data in Section 4.2.1, the outside diameter is somewhat less. This is to avoid the necessity of cutting the gear with shaper cutters that have excessively pointed teeth. The limit set is top land with $0.40/P_d$. Table 4.35 gives the tooth proportions for face gear pinions.

**Face gear design.** The face-gear member is generated by a cutter having proportions based on the pinion with which the face gear will operate. The most important specification for the shape of the teeth of a face gear is a complete specification of the cutter to be used to cut it or, next best, a detailed specification of the mating pinion.

The face gear has two dimensions unique to face gears which control the face width of the teeth: the outer and inner diameter of the face gear.

---

**TABLE 4.32**

Tooth Proportions for Pinions Meshing with Face Gears

<table>
<thead>
<tr>
<th>Item</th>
<th>Coarse Pitch, $P_d \leq 20$</th>
<th>Fine Pitch, $P_d &gt; 20$</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of teeth in pinion, $N_p$</td>
<td>$N_p &lt; 18$</td>
<td>$N_p \geq 18$</td>
</tr>
<tr>
<td>Pressure angle, $\phi$, deg</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>Addendum, $a$</td>
<td>See Table 4.34</td>
<td>$1/P_d$</td>
</tr>
<tr>
<td>Standard pitch diameter, $D_p$</td>
<td>$N/P_d$</td>
<td>$N/P_d$</td>
</tr>
<tr>
<td>Working depth, $h_k$</td>
<td>$2/P_d$</td>
<td>See Table 4.35</td>
</tr>
<tr>
<td>Whole depth, $h_t$</td>
<td>$2.25/2P_d$</td>
<td>$\pi/2P_d$ + 0.002</td>
</tr>
<tr>
<td>Circular tooth thickness*, $t$</td>
<td>$\pi/2P_d$</td>
<td>See Table 4.35</td>
</tr>
<tr>
<td>Clearance, $c$</td>
<td>$0.25/P_d$</td>
<td>$0.2/P_d + 0.002$</td>
</tr>
</tbody>
</table>

* Thin teeth for backlash. See Section 4.5 for general information on spur gear backlash
### TABLE 4.33
Minimum Numbers of Teeth in Pinion and Face Gear

<table>
<thead>
<tr>
<th>Diametral Pitch Range</th>
<th>Min. * Numbers of Teeth</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Pinion</td>
</tr>
<tr>
<td>20 – 48</td>
<td>12</td>
</tr>
<tr>
<td>49 – 52</td>
<td>13</td>
</tr>
<tr>
<td>53 – 56</td>
<td>14</td>
</tr>
<tr>
<td>57 – 60</td>
<td>15</td>
</tr>
<tr>
<td>61 – 64</td>
<td>16</td>
</tr>
<tr>
<td>65 – 68</td>
<td>17</td>
</tr>
<tr>
<td>69 – 72</td>
<td>18</td>
</tr>
<tr>
<td>73 – 76</td>
<td>19</td>
</tr>
<tr>
<td>77 – 80</td>
<td>20</td>
</tr>
<tr>
<td>81 – 84</td>
<td>21</td>
</tr>
<tr>
<td>85 – 88</td>
<td>22</td>
</tr>
<tr>
<td>89 – 92</td>
<td>23</td>
</tr>
<tr>
<td>93 – 96</td>
<td>24</td>
</tr>
<tr>
<td>97 – 100</td>
<td>25</td>
</tr>
</tbody>
</table>

* The minimum numbers of teeth in the pinion are limited by the design requirements of the gear cutter. These requirements, in addition to the minimum-gear-ratio limitation, limit the numbers of teeth in the gear.

### TABLE 4.34
Addendum of Face Gears and Pinions

<table>
<thead>
<tr>
<th>No. of Teeth in Pinion</th>
<th>Coarse Pitch</th>
<th>Fine Pitch Pinion Addendum</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Pinion Addendum</td>
<td>Gear Addendum</td>
</tr>
<tr>
<td>12</td>
<td>1.120</td>
<td>0.700</td>
</tr>
<tr>
<td>13</td>
<td>1.100</td>
<td>0.760</td>
</tr>
<tr>
<td>14</td>
<td>1.080</td>
<td>0.820</td>
</tr>
<tr>
<td>15</td>
<td>1.060</td>
<td>0.880</td>
</tr>
<tr>
<td>16</td>
<td>1.040</td>
<td>0.940</td>
</tr>
<tr>
<td>17</td>
<td>1.020</td>
<td>0.980</td>
</tr>
<tr>
<td>18 and 19</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>20 and 21</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>22 through 29</td>
<td>1.000</td>
<td>1.000</td>
</tr>
</tbody>
</table>

(Continued)
The maximum useable face width may be estimated from Table 4.36. The outside diameter of the gear should not exceed \( m_o D \). The inside diameter should not be less than \( m_i D \). Thus:

\[
\text{Outer diameter} = m_o D = D_o \quad (4.47)
\]
\[
\text{Inner diameter} = m_i D = D_i \quad (4.48)
\]
\[
\text{Pitch diameter} = D \quad (4.49)
\]
\[
\text{Face width} = \frac{D_o - D_i}{2} \quad (4.50)
\]

### 4.2.7 System for Spiroid and Helicon Gears

At the present time there are no standards covering Spiroid, Helicon, or Planoid\(^{14}\) gears. The following material is based on information covered in “Spiroid Gearing”, paper 57-A-162 of the ASME, and on additional information supplied by its author, W.D. Nelson.

\(^{14}\) Spiroid, Helicon, and Planoid are trademarks registered by the Illinois Tool Works, Chicago, IL.
Although specialized designs can be developed that will best suit a given application, the standardized procedure presented here will yield designs that will meet the needs of most Spiroid and Helicon gear applications.

Planoid gears are often associated with Spiroid and Helicon gearing but are an entirely different form of gearing, used for ratios generally under 10:1 where maximum strength and efficiency are required. Their design is beyond the scope of this section.

The basic approach to the establishment of the Spiroid or Helicon tooth form is to establish the pinion tooth form and then develop a gear tooth form that is conjugate to it.

The tooth form of the Spiroid gear is compromised of teeth having different pressure angles on each side of the teeth: a “low”-pressure-angle side and a “high”-pressure-angle side. The choice of pressure angle will control the extent of the fields of conjugate action of the teeth. The values of pinion taper angle have been standardized by the Illinois Tool Works in order to achieve the best general-purpose gearing. These standard values are shown in Table 4.37.

The design of Helicon gearing is closely related to Spiroid gearing. Table 4.38 shows the general equations used in the design of both kinds of gearing. The general Spiroid formulas become Helicon formulas when the pinion taper angle $\tau$ is set equal to zero. Figure 4.14 shows a Spiroid gear set. A Helicon pinion and gear are similar except that the pinion taper angle $\tau$ and the gear face angle $\gamma$ are both zero in Helicon gearing.

**Spiroid gearing.** Spiroid gearing is suitable for gear ratios of 10:1 or higher. The numbers of teeth in the gear can range from 30 to 300.

### Table 4.36

**Tooth Proportions and Diameter Constants for 1 Diametral Pitch Face Gears, 20° Pressure Angle**

<table>
<thead>
<tr>
<th>No. of Pinion Teeth, $N_p$</th>
<th>Gear Diameter Constraints</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$m_g = 1.5$</td>
</tr>
<tr>
<td></td>
<td>$m_o$, $m_i$</td>
</tr>
<tr>
<td>12</td>
<td>...</td>
</tr>
<tr>
<td>13</td>
<td>1.202, 1.064</td>
</tr>
<tr>
<td>14</td>
<td>1.187, 1.062</td>
</tr>
<tr>
<td>15</td>
<td>1.174, 1.052</td>
</tr>
<tr>
<td>16</td>
<td>1.161, 1.051</td>
</tr>
<tr>
<td>17</td>
<td>1.156, 1.041</td>
</tr>
<tr>
<td>18</td>
<td>1.150, 1.039</td>
</tr>
<tr>
<td></td>
<td>1.176, 1.042</td>
</tr>
<tr>
<td>20</td>
<td>1.144, 1.030</td>
</tr>
<tr>
<td></td>
<td>1.166, 1.032</td>
</tr>
<tr>
<td>22</td>
<td>1.140, 1.022</td>
</tr>
<tr>
<td></td>
<td>1.156, 1.024</td>
</tr>
<tr>
<td>24</td>
<td>1.133, 1.015</td>
</tr>
<tr>
<td></td>
<td>1.150, 1.017</td>
</tr>
<tr>
<td>30</td>
<td>1.121, 1.001</td>
</tr>
<tr>
<td></td>
<td>1.131, 1.001</td>
</tr>
<tr>
<td>40</td>
<td>1.109, 0.986</td>
</tr>
<tr>
<td></td>
<td>1.113, 0.986</td>
</tr>
</tbody>
</table>
TABLE 4.37
Standard Tooth and Gear Blank Relationships, Spiroid and Helicon Gearing

Spiroid gearing:
1. “Sigma” angle, \( \varphi_p = 40^\circ \) (standard)
2. Pinion taper angle \( \tau \) vs. gear face angle \( \gamma \)

<table>
<thead>
<tr>
<th>Pinion taper angle, ( \tau ), deg</th>
<th>Gear face angle*, ( \gamma ), deg</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>8 preferred</td>
</tr>
<tr>
<td>7</td>
<td>11</td>
</tr>
<tr>
<td>10</td>
<td>14</td>
</tr>
</tbody>
</table>

3. Gear ratio \( m_G = 10:1 \) higher
4. Number of teeth in gear:
   Varies with center distance (see Table 4.18)
   Hunting ratios are desirable with all multiple-thread pinions
5. Tentative pressure angle selection

<table>
<thead>
<tr>
<th>Ratio</th>
<th>Pressure angle selection</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m_G \leq 16:1 )</td>
<td>Low side</td>
</tr>
<tr>
<td></td>
<td>15°</td>
</tr>
<tr>
<td>( m_G &gt; 16:1 )</td>
<td>10°</td>
</tr>
</tbody>
</table>

* Approximate values, for \( \varphi_p = 40^\circ \).

TABLE 4.38
General Equations for Spiroid and Helicon Gearing

Spiroid Gearing:

\[
\sin \phi_p = \frac{\tan \tau}{\tan \varphi_p} \tag{4.51}
\]

\[
R_p = \frac{C}{\sin \phi_p + \frac{G}{R_p} \cos \varphi_p} \tag{4.52}
\]

\[
R_G = \frac{R_p}{R_p} \tag{4.53}
\]

\[
L = \frac{2}{m_G - \frac{G}{R_p}} \sin \varphi_p \cos \phi_p \tag{4.54}
\]

\[
x_p = R_G \sin \varphi_p \tag{4.55}
\]

\[
r_o = R_p - x_p \tan \tau \tag{4.56}
\]

\[
D_W = \frac{0.6 \frac{L}{R_p} \sec \tau}{\sin \frac{\phi_1}{\cos (\phi_1 + \tau)} + \sin \frac{\phi_2}{\cos (\phi_2 - \tau)}} \tag{4.57}
\]

\[
D_N = D_W \cos \tau \tag{4.58}
\]

\[
CLR = 0.07 \frac{L}{x_p} + 0.002 \tag{4.59}
\]

\[
\psi_{2L} = \tan^{-1} \left[ \frac{\frac{G}{x_p} + \frac{C}{R_p} \frac{x_p}{x_p - x_p}}{\frac{x_p}{x_p - x_p}} \right] \tag{4.60}
\]

\[
\tan \lambda_m = \frac{L \sec \tau}{2 \gamma_m} \tag{4.61}
\]

\[
\psi_2 = \psi_{2L} + 5^\circ \tag{4.62}
\]

(Continued)
TABLE 4.38 (Continued)
General Equations for *Spiroid* and *Helicon* Gearing

*Spiroid* Gearing:

Helicon gearing:

\[
R_p = \frac{C}{\frac{R_G \cos \beta_p}{R_p}} \quad (4.63)
\]

\[
L = \frac{2 \cdot R_G \cos \beta_p}{m_G - \frac{R_G}{R_p} \sin \beta_p} \quad (4.64)
\]

\[
D_W = \frac{0.6 \cdot L}{\tan \psi_1 + \tan \psi_2} \quad (4.65)
\]

\[
\psi_{2L} = \tan^{-1} \frac{R_p}{x_p} \quad (4.66)
\]

\[
\psi_{2L} = \tan^{-1} \frac{1}{\frac{R_G}{R_p} \sin \beta_p} \quad (4.67)
\]

\[
\psi_2 = \psi_{2L} + 7^\circ \quad (4.68)
\]

FIGURE 4.14 *Spiroid* Gear Set—Mounting and Gear Blank Nomenclature.

The design procedure for *Spiroid* gearing differs somewhat from other types of gearing discussed in this chapter. The following are the basic steps:

- Center-distance is the basic starting point for the calculation of *Spiroid* tooth proportions
- Lead and pinion “zero” plane radius are then calculated
- The tooth proportions and the blank proportions are based on the pinion zero plane radius and are then calculated

From this procedure it can be seen that the pitch of the teeth is an end result of the calculating procedure for *Spiroid* gearing, rather than the beginning as in other types of gearing. Center-distance governs the horsepower capacity of the gear set.
Table 4.37 shows the basic tooth proportions for Spiroid gearing. Table 4.38 gives the general equations used to determine Spiroid tooth proportions.

If optimum efficiency is desired, the size of the pinion relative to the gear should be kept small by using values of $R_G/R_P$ which are larger than those shown in Figure 4.15. This has the effect of producing an increased lead angle for a given center-distance and gear ratio. Figures 4.16 and 4.17 show recommended data for Helicon and Spiroid calculations.

If control gear applications being considered, in which runout is critical, the driving side should be the low-pressure-angle side of the teeth. This minimizes the effects of runout in both members. A $10^\circ$ taper angle $\tau$ may be used, since it requires only one-half the axial movement of the pinion to produce a given change in backlash.

For gear sets of the highest strength, the number of teeth in the gear should be kept low (30 to 40 for lower ratios) and a stub tooth form employed. Such designs are beyond the scope of this section.

**FIGURE 4.15** Gear Tooth High Point—Spiroid Gearing.

**FIGURE 4.16** Sigma Angle for Different Ratios—Helicon Gearing.
Straddle-mounted pinions may require a shaft of larger size (greater stiffness) than would result from the data shown in Figure 4.14. In such cases, the values of \( R_G, R_P \) are decreased until a shaft of the desired size can be achieved. There is a definite limit to the maximum size of pinion for a given pressure angle. The size of the pinion may be increased until a “limiting pressure” condition is reached. Limiting pressure angle can be calculated by means of Eq. (4.62)

**Helicon gearing.** Helicon gearing is suitable for gear ratios from 4:1 to 400:1 in light- to medium-loaded applications.

The design procedure for Helicon gears is somewhat similar to that for Spiroid gears but starts with the gear outside diameter as the basis of load capacity instead of the center-distance as is used in Spiroid gearing.

The following are the basic steps:

- The gear ratio \( m_G \), pinion \( r/\text{min} \), and horsepower are used to establish the necessary gear outside diameter
- Pinion pitch radius, center-distance, lead, and working depth are next established
- From these values, the gear and pinion blank dimensions are established

### 4.3 GENERAL EQUATIONS RELATING TO CENTER-DISTANCE

This section of Chapter 4 deals with the distance between the shafts of meshing gears. This distance is called *center-distance* in the case of gearing operating on nonintersecting shafts. Spur and helical gearing, both external and internal, worm gearing, hypoid gearing,\(^{15}\) spiral gearing, Spiroid\(^ {16} \) gearing, and Helicon\(^ {17} \) gearing must all operate at specific center-distances. Certain types of gears which operate on intersecting shafts, such as bevel gearing, do not have a center-distance dimension. However, their pitch surfaces must be maintained in the correct relationship; hence, the axial position of these gears along their shafts is critical. Such gears, therefore, have a *mounting distance* that defines the axial position of the gears.

---

\(^{15}\) In the case of hypoid gearing, *offset* is the correct term for the distance between gear and pinion shafts.

\(^{16}\) Spiroid is a trademark of the Illinois Tool Works, Chicago, IL.

\(^{17}\) Helicon is a trademark of the Illinois Tool Works, Chicago, IL.
Certain types of gears, such as hypoid gears, Spiroid gears, throated worms and worm gears, face gears, Helicon gears, and Planoid\textsuperscript{18} gears, have both a center distance and a mounting distance.

When establishing the center-distance for a set of gears, it is customary first to determine the theoretical center-distance for the gears. The actual operating center-distance is next determined. This center-distance includes considerations of tolerance and the interchangeability of the various parts that may be included in the final assembly such as bearing, mounting brackets, and housing.

In spur and helical gears, it may be necessary or desirable to operate the gears on an appreciably larger center-distance than theoretical to get improved load-carrying capacity. For instance, gears can be designed so that 20° hobs cut the tooth; but the “spread” center is calculated so as to make the gears operate at a 25° pressure angle. See Sections 4.3.4, 4.3.5, and 4.3.6.

The next section covers the calculation of the theoretical center-distance at which various types of gearing will operate. The most generally used equations are summarized in Table 4.39. In the Sections that follow, the special consideration on which the equations are based are covered in detail.

### 4.3.1 Center-Distance Equations

Table 4.39 is a summary of equations convenient to use to obtain the values of center-distance for the various types of gearing shown in Figure 4.18.

Center-distance and tooth thickness are inseparable. In most case, however, standard tooth proportions are used, and the simplified equations of center-distance [Eq. (4.69) through Eq. (4.72)] are adequate. Standard tooth proportions are defined in Section 4.3.2. When the sum of the tooth thickness of pinion and gear does not equal the circular pitch, nonstandard center-distance, as obtained from equations such as Eq. (4.73) and Eq. (4.76), will be required. Most problems involving center distance and tooth thickness fall into one of the following categories:

- The thickness of the teeth of the gear and of the pinion is fixed. The center-distance at which the gears will mesh properly is to be established
  a. The sum of the tooth thicknesses of both members plus the design backlash is equal to the circular pitch. In this case, standard center-distance is correct. Use Eq. (4.69) and Eq. (4.72)
  b. The sum of the tooth thickness of both members plus the design backlash is not equal to the circular pitch. In this case, a nonstandard center-distance is required. Use Eq. (4.73) and Eq. (4.76). This problem is covered in greater detail in Section 4.3.9.

- The center-distance is fixed. The tooth thicknesses for gears that will operate on the given center-distance are to be established. If the center-distance for the given diametral pitch and number of teeth is different from that obtained from Eq. (4.70), then Eq. (4.73) must be solved and the operating pressure angle Thus, found used in Eq. (4.76), which is then solved for the term \((t_p + t_g + B)\). In Section 4.1.4, the ways in which this total tooth thickness can be divided between gear and pinion are discussed

- Neither the tooth thickness nor the center-distance is fixed; the best values for both are to be established. In this happy case, the tooth thickness is usually established first on the basis of strength or some other appropriate consideration and the center distance required is then based on Eq. (4.76) and Eq. (4.73)

- Both the center-distance and the tooth thickness are fixed. The amount of backlash or the degree of tooth interference is to be established. This is a frequent “check problem” of an existing gear design. Equations (4.73) and (4.76) are used and solved for \(B\). Minus values of \(B\) indicate tooth interference

\textsuperscript{18} Planoid is a trademark of the Illinois Tool Works, Chicago, IL.
TABLE 4.39  
Center-Distance Equations

**Standard Center-Distance, \( C \):**

- Spur, helical, and worm gears
  \[
  C = \frac{d + D}{2} 
  \]  
  \[
  C = \frac{n + N}{2P} 
  \]  
- Internal gears
  \[
  C = \frac{D - d}{2} 
  \]  
  \[
  C = \frac{N - n}{2P} 
  \]

**Operating center-distance, \( C' \):**

- Spur, helical, and worm gears
  \[
  C' = \frac{C}{\cos\phi\cos\phi'} 
  \]  
  \[
  C' = \frac{d + D'}{2} 
  \]
- Spur helical
  \[
  C' = \frac{Dd}{2} - \frac{Nn}{P} 
  \]
- Internal
  \[
  C' = \frac{D' - d'}{2} 
  \]

General equations relating tooth thickness and center-distance for parallel axis gearing:

- External spur and helical gears
  \[
  \text{inv } \phi' = \frac{n (p + y + B) - nd}{(n + N) d} + \text{inv } \phi 
  \]  
- Internal gears
  \[
  \text{inv } \phi' = \frac{nd - n (p + y + B)}{(N - n) d} + \text{inv } \phi 
  \]

General equations relating tooth thickness and center-distance for nonparallel nonintersecting axis gearing

- \( K_a = \frac{n}{N} = \frac{1}{m_G} \) (4.78)
- \( K_b = \text{inv } \phi_G + K_a \text{inv } \phi_p = \frac{n (p - y - B - a_0)}{N p_s} \) (4.79)
- \( K_d = \frac{\text{inv } \phi_d}{\text{inv } \phi_p} \) (4.80)
- \( K_c = K_a + K_d \) (4.81)
- \( \text{inv } \phi_p = \frac{K_b}{K_c} \) (4.82)
- \( \text{inv } \phi_G = K_b - K_a \text{inv } \phi_p \) (4.83)
- \( \text{inv } \phi_G' = \frac{\text{inv } \phi_G}{\text{inv } \phi_p} \) (4.84)
- \( K_{d'} = K_d \) (check equation) (4.85)
- \( D'P = \frac{b_0 p}{\cos\phi_p} \) (4.86)
- \( D'G = \frac{b_0 G}{\cos\phi_G} \) (4.87)

Use values from Eqs. (4.84) and Eq. (4.85) in Eq. (4.74), where

\[
K_c = \frac{\sin\phi_G}{\sin\phi_p} \frac{\sin\phi_G'}{\sin\phi_p} 
\]

- \( P \) – transverse diametral pitch
- \( C \) – “standard” center-distance [defined by Eq. (4.69) and Eq. (4.70)]. See also Section 4.13
- \( C' \) – operating center-distance
- \( d \) – “standard” \((n/p)\) pitch diameter of pinion. See also Section 4.14
- \( d' \) – operating pitch diameter of pinion. See also Section 4.15
- \( D \) – “standard” \((N/P)\) pitch diameter of gear. See also Section 4.14
- \( D' \) – operating pitch diameter of gear. See also Section 4.15
- \( n \) – number of teeth in pinion
- \( N \) – number of teeth in gear
- \( \phi \) – cutting pressure angle
- \( B \) – design backlash, the total amount the teeth in both members are thinned for backlash considerations
4.3.2 **STANDARD CENTER-DISTANCE**

Most gear designs are based on standard tooth proportions. Such gears are intended to mesh on standard center-distances. The equations that establish standard center-distances are based on the following assumptions:

1. The sum of the circular tooth thicknesses (effective) equals to the circular pitch minus the backlash:

   \[ t_p + t_g = p_t - B \]  \hspace{1cm} (4.89)

   This rule covers the case of long and short addendums in which the gear teeth are corrected for such things as undercut or balanced strength. In such cases, the tooth thicknesses of both members are altered sufficiently to permit to mesh on standard center-distances.

2. The tooth thickness of an individual gear is one-half the circular pitch (transverse) minus one-half the backlash:

   \[ t = \frac{p - B}{2} \]  \hspace{1cm} (4.90)

   This covers the more common case. It represents the approach used by most makers of “catalogue”-type gears. Such gears are all expected to operate on standard center-distances regardless of the number of teeth.
teeth in the pinion. The center-distances on which such gears will operate may be calculated by means of Eq. (4.69) and Eq. (4.70).

The tolerance on standard center-distance can be bilateral (Thus, 5.000 in. ± 0.002 in.) if the value of B was chosen sufficiently large. This is the most convenient way from the standpoint of manufacturer. If B is not large enough, a unilateral tolerance (Thus, 5.000 in. + 0.004 in. − 000 in.) may be used.

4.3.3 **Standard Pitch Diameters**

The standard pitch diameter of a gear is a dimension of a theoretical circle. It is given for each type of gear by the following relations:

**Spur gearing:**

\[
\frac{N}{P_d}
\]

(4.91)

where \(P_d\) is the diametral pitch.

**Helical gearing:**

\[
\frac{N}{P_t}
\]

(4.92)

where \(P_t\) is the transverse diametral pitch.

**Bevel gearing:**

\[
\frac{N}{P_d}
\]

(4.93)

**Worm gearing:**

\[
\frac{N_G p}{\pi}
\]

(4.94)

where \(N_G\) equals the number of worm gear teeth, and \(p\) equals the circular pitch of the worm gear. Since the diametral pitch of a gear is fixed by the tool that is used to cut it (hob, shaper cutter, shaving cutter, and so forth), and since the number of teeth in the gear is a whole number, the *standard* pitch diameter is an imaginary circle. It can have no tolerance, regardless of the variations in tooth thickness or in the center-distance on which it is to operate.

4.3.4 **Operating Pitch Diameters**

As shown elsewhere, it is entirely practical to operate involute gears of specific diametral pitch and numbers of teeth at various center-distances. It is convenient in such cases to calculate *operating* pitch diameters for such gears. See (4.95) through (4.98).

**External spur and helical pinions:**

\[
d' = \frac{2C'}{m_G + 1}
\]

(4.95)
Internal spur and helical pinions:

\[ d' = \frac{2 C'}{m_G - 1} \]  

(4.96)

External spur and helical gears

\[ D' = \frac{2 C'}{m_G + 1} \]  

(4.97)

Internal spur and helical gears:

\[ D' = \frac{2 C'}{m_G - 1} \]  

(4.98)

where:

- \( C' \) – is the operating center-distance
- \( m_G \) – is the gear ratio, \( \left( \frac{N}{n} \right) \)
- \( d' \) – is the operating pitch diameter of pinion
- \( D' \) – is the operating pitch diameter of gear

These equations define the operating pitch diameters as being proportional to the transmitted ratio and the instantaneous center-distance.

Since gears with thicker-than-standard teeth must operate on enlarged center-distances, they will run on operating pitch diameters that are larger than their standard pitch diameters. The operating pitch diameter should not be specified as a drawing dimension on the detail drawing of the gear since it will vary for ever different gear and center-distance. The best place to show the operating center-distance is on an assembly drawing that shows both the pinion and gear that mesh together.

### 4.3.5 Operating Pressure Angle

The pressure angle of an individual gear is based on the diameter of the base circle of the gear and on the identification of the specific radius at which the pressure angle is to be considered. In standard gears, this radius is customarily the standard pitch radius. It is convenient to consider the pressure angle of gears operating on nonstandard center-distances at the point of intersection of the line of action and line of centers. This is the definition of the operating pressure angle which may be calculated by means of (4.99):

\[ \cos \phi' = \frac{C}{C'} \cos \phi \]  

(4.99)

where:

- \( \phi' \) – is the operating pressure angle
- \( C \) – is the standard center-distance [see Eq. (4.70)]
- \( C' \) – is the operating center-distance
- \( \phi \) – is the standard pressure angle
It can be seen from the foregoing that a gear can be cut with a cutter of one pressure angle and operate at a different pressure angle. This flexibility causes much of the confusion in gear design. It is necessary to define accurately each of the standard elements of a nonstandard gear—pitch and pressure angle. A specification of base-circle diameter is highly desirable.

4.3.6 Operating Center-Distance

The actual center-distance at which a gear will operate will have a large influence on the way in which the gear will perform in service. The actual operating center distance is made up of the combined effects of manufacturing tolerances, the basic center distance, differential expansions between the gears and their mountings, and deflections in the mountings due to service loads.

The items that should be considered when determining the minimum and maximum operating center-distance for any given gear design are discussed in detail in Section 4.3.9 and Section 4.4.1.

In any critical evaluation of a gear design, particularly in the field of control gearing, the minimum and the maximum operating center-distance should be used in equations covering backlash, contact ratio, tooth tip clearance, and so forth.

In this chapter the concept operating center-distance is used in two ways. In one case, it is considered to be the center-distance that results from the buildup of all tolerances that influence center-distance in any one case. This concept is illustrated in case II in Section 4.4.1. Thus, it is the largest or smallest actual center-distance that could be encountered in a given design. In the other case, the one covered by Eq. (4.73) and Eq. (4.76). For example, it is the center-distance at which gears of a specified tooth thickness and backlash will operate. Depending on the problem encountered, the correct concept to use will have to be selected by the designer.

4.3.7 Center-Distance for Gears Operating on Nonparallel Nonintersecting Shafts

The most frequently encountered example of this type of gear operation is in the shaving or lapping of gears. Here, a cutter, usually a helical-toothed member, is run in tight mesh with a spur or helical gear. The operating center-distance is dependent on the tooth thickness of each member. Equations (4.78) through Eq. (4.87) should be solved in sequence and the result used in Eq. (4.73) to obtain the operating center-distance. Equation (4.88) gives the ratio constant for the transverse pressure angles for crossed axis gearing.

4.3.8 Center-Distance for Worm Gearings

The center-distance for worm gearing is based on the sum of the standard pitch diameters of the worm and worm gear [(4.100)]. The standard pitch diameter of the worm is obtained from (4.101). This circle has no kinematic significance but is the basis for worm tooth proportions:

\[
C = \frac{d_w + D}{2} \quad (4.100)
\]

\[
d_w = \frac{L}{\pi \tan \lambda} \quad (4.101)
\]

where:

- \(C\) is the center-distance
- \(d_w\) is the pitch diameter of worm
- \(D\) is the pitch diameter of gear
L – is the lead
λ – is the lead angle

4.3.9 REASONS FOR NONSTANDARD CENTER-DISTANCES

On some occasions a set of gears must operate on a center-distance that is not one-half the sum of the standard pitch diameter of the meshing gears. The designer is confronted with nonstandard center-distance in several situations, the more important of which are:

- Gear trains in which the teeth are made to standard tooth thickness and backlash is introduced by increasing the standard center-distance slightly
- Gear trains in which the number of teeth and pressure angle relationship requires a long addendum (enlarged tooth thickness design) to avoid undercut, and yet the number of teeth in the gear is so small that to make the gear sufficiently short addendum to compensate for the pinion enlargement would cause undercut. In such cases, an enlarged center-distance is usually indicated
- Gear trains in which the sum of the tooth thicknesses of pinion and gear is not equal to the circular pitch for reasons of tooth strength, wear, or scoring
- Gear trains in which a minor change in ratio (total number of teeth in mesh) has been made without a change in center-distance

In each of these cases, the calculation of operating center-distance is performed using Eq. (4.73) and Eq. (4.76), or Eq. (4.77).

4.3.10 NONSTANDARD CENTER-DISTANCES

By proper adjustments of the thickness of the teeth on each gear of a meshing pair, it is possible to achieve gear designs that will meet most nonstandard center-distances. In Section 4.1.3, the limitations governing tooth thickness are outlined. In cases where maximum strength is not critical, it will be found that gears not exceeding these limitations will satisfy most nonstandard center-distance problems encountered. This is discussed more fully in method 3 below.

Sometimes designers who do not realize the possibilities of gear design will select such things as fractional diametral pitches (10.9652, etc.) in order to make gears fit a nonstandard center-distance. This is poor practice is that it necessitates special tooling, which may be costly. It is better to attempt to utilize standard-pitch tooling and, by proper adjustments of tooth thickness, to establish gear designs that will meet the nonstandard distances.

In cases wherein the center-distance is nonstandard, the designer has four possible methods of designing gears to meet the given center-distance:

1. A number of teeth in gear and pinion different from the numbers originally selected may be found which will more nearly meet the given center-distance. A Brocot table is very useful in finding the numbers of teeth that will give different gear ratios \( l/m_G \). In this case, it is assumed that any of several ratios may be “good enough”. If not, the following alternatives, which all assume that a specific ratio must be obtained, may be considered

2. Equation (4.69), Eq. (4.70), Eq. (4.71), or Eq. (4.72) may be rearranged to solve for \( P_d \). Thus, a diametral pitch that will meet the gear ratio and center-distance is found. This method will yield nonstandard diametral pitches for spur gears, which will almost always lead to large manufacturing costs because of the need for nonstandard tooling, cutters, and master gears. This method should be used only if methods 2, 3, and 4 do not meet the needs of application
3. The gear and pinion can be made with teeth that are thicker or thinner than standard. The easiest approach when using this method is to follow method 2 above, and then use the results as the basis for a selection of the standard diametral pitch nearest to the one found. Equation (4.73) is then used, which will yield the operating pressure angle for the gears based on the new selection of diametral pitch and the desired center-distance \( C' \). The value of \( C \) in this equation is determined from Eq. (4.69) or Eq. (4.70). Equation (4.76) is then used and solved for the term \( (t_p + t_G) \). The selection of values for each term, \( t_p \) and \( t_G \), should be based somewhat on the ratios of their number of teeth. In general, pinions cannot be changed from standard as much as gears with large numbers of teeth. The selected value of tooth thickness should be checked for undercut in the event that \( t_p \) is less than \( p/2 \). If the thickness is appreciably greater than \( p/2 \) the pinion should be checked for pointed teeth. The same should be done for the gear, especially if the changed tooth thickness was not based on the gear ratio. Once the values of \( C' \), \( P \), \( n \), \( N \), \( t_p \), and \( t_G \) have been established, the remainder of the tooth proportions can be calculated from these values. If the addendum and whole depth are properly adjusted from the tooth thickness, standard tools can be used to cut and to inspect these gears.

4. If helical gears can be used, the helix angle can be adjusted to obtain the required tooth thickness-center distance relationship. In effect, the designer is following method 3 above but is adjusting the transverse tooth thickness by the selection of the required helix angle. The procedure in this case is to use Eq. (4.70) or Eq. (4.72) to establish the diametral pitch required in the transverse plane \( P \) to suit given center-distance. The nearest finer (smaller) diametral pitch for which tools are available is selected \( (P_n) \), and these values are used in (4.102) to establish the required helix angle:

\[
\cos \psi = \frac{P}{P_n} \tag{4.102}
\]

Although not an essential part of this problem, somewhat smoother-running gears will result if the designer can manage to pick diametral pitches that will result in a helical overlap of at least 2. After the helix angle has been found, the designer can determine the remaining tooth proportions from data found in this chapter. This method can be used economically if the hobbing process of cutting gears is to be used. Special guides and shaper cutters are required for every different lead of helical gear for gears cut by the shaping process. This then fixes limits on the values of helix angle that any given shop could cut.

### 4.4 ELEMENTS OF CENTER-DISTANCE

Up to this point, this chapter has considered only the means of determining the theoretical center-distance that is required by a set of gears. The theoretical center-distance is either the basic center-distance, as established by Eq. (4.69), Eq. (4.70), Eq. (4.71), or Eq. (4.72), or the theoretical tight-mesh center-distance, as established by Eq. (4.73) through Eq. (4.88). In an actual gear set, however, the gears will be forced to operate on a real center-distance that may be larger or smaller than the theoretical center-distance by amounts that are based on the way that the part tolerances (bearings, casing, bores, and so forth) will add up.

#### 4.4.1 EFFECTS OF TOLERANCES ON CENTER-DISTANCE

In all but the very simplest gear casing designs, there are many tolerances that will govern the operating position of the gear and pinion shafts. Usually, power gearing is not designed to small values of backlash. It is customary, therefore, to apply a generous amount of backlash to the gear teeth, a value that has been found by experience sufficient to prevent binding of the teeth. In control gearing, however, particularly in
those applications that are operated in both directions, and are intended to have a minimum of lost motion, a critical study of the effects of part tolerances is usually required. The tooth thickness can then be made as large as possible, ensuring a minimum of backlash with little risk of binding. The following two examples indicate a very simple case and a more complex case of center-distance calculation.

Case I. In Figure 4.19 is shown a spur gear pair mounted on a cast-iron gear case. The loads and speeds are such that the bearings are simply smooth bores in the cast iron with shafts running in them. Figure 4.20 is a vector diagram showing the forces produced on the bearings as a result of the gear tooth loads. Figure 4.21 shows the displacements of the shafts in the bearing resulting from these loads.

In this example, it is desired to establish the largest and smallest center-distance that the gears will ever experience in order to determine the range of backlash existing in the gear set. Table 4.40 outlines the calculations made to establish the various center-distances at which the gears may be expected to operate in service. Part A of Table 4.40 shows the calculation of the maximum distance that the axis of either shaft can move away from the axis of its bore. This is given as the eccentricity of the shaft within the bearing. It is assumed that the bearing is dry. For a very rough approximation of maximum and minimum possible center-distances, these values of eccentricity can be added to and subtracted from the boring center-distance plus its tolerance. This is shown in part B of Table 4.40. This assumes the forces acting on the shafts are tooth-load reactions that act along line of centers. In the case of journal bearings that are properly lubricated, the calculated value of eccentricity of operation can be used. This calculation is discussed in texts on journal bearings design. In most cases if the designer can select the tolerances for the gear tooth thickness, and also the tolerances on the various parts of the casing and bearings that control the center-distance, such that the backlash is not excessive when the calculations shown thus far

![Figure 4.19](image-url)  
**FIGURE 4.19** Spur or Helical Gears Operated in Bearings Bored in the Gear Casing Exhibit the Simplest Case of Tolerance Buildup on Center-Distance.
are carried out, no further calculations need be made. However, when the design is to achieve an absolute minimum of lost motion, the more accurate evaluation of center-distance should be carried out.

Part C shows a calculation assuming that the gear tooth reactions will control the position of the journal in the bearing. The minimum center-distance will occur when the minimum boring center-distance is achieved and the minimum bearing clearance occurs. The tooth loads tend to increase the center-distance in this example so that less than the minimum boring center-distance cannot occur.

In actual service, the maximum and minimum operating center-distances will rarely reach these limits. In the case of maximum center-distance, a properly lubricated shaft will develop an oil wedge that keeps the journal away from the walls of the bore. Second, if the only separating loads on the shafts are those produced by gear tooth reactions, the radial movement of the shaft along the line of centers will be equal $e \cdot (\sin \Phi)$. The proper evaluation of operating center-distance will depend on whether the application is intermittent in rotation and on the direction of rotation.

Case II. A more complex case occurs when the pinion and gear shafts are mounted in separate units. The following example illustrates a case in which a motor having a pinion cut integral with the shaft is mounted by means of a rabbet in a gear case. This type of construction is often found in power hand tools and in aircraft actuators. This example also illustrates a critical evaluation of antifriction bearings. It is desired to determine the largest and smallest center-distance that can occur in order to establish the maximum and minimum tooth thickness and the amounts of backlash that result (see Figure 4.22).

Table 4.41, which summarizes the calculations, shows a total of 16 different elements that combine to control the maximum and minimum actual center-distance. Since there are so many tolerances, the use of the root mean square of these values is suggested as a reasonable approximation to operating conditions,
ΔC – is increase in center distance due to movement of journal in bearing; e – is eccentricity of bearing

\[ e = \frac{\text{bore diameter} - \text{journal diameter}}{2} \]

**FIGURE 4.21** The Position Occupied by the Journal in a Bearing Is a Function of the Gear Tooth Loads along the Shaft and the Effects of Friction or of the Oil Wedge in the Bearing.

---

**TABLE 4.40**

**Summary of Calculations for Case I**

* A. Calculation of bearing clearance and shaft eccentricity. See Figure 4.19 for drawing dimensions

| Bore for pinion shaft | Max. 1.0010 | Min. 1.0000 |
| Bore for gear shaft | Max. 1.5010 | Min. 1.5000 |

| Pinion-shaft diameter | Min. 0.9992 | Max. 0.9997 |
| Gear shaft diameter | Min. 1.4992 | Max. 1.4997 |

| Clearance in bearing (c) | Max. 0.0018 | Min. 0.0003 |
| Eccentricity of shaft position in bearing (c/2) | Max. 0.0009 | Min. 0.00015 |

| Eccentricity of pinion bearing | 0.0009 | -0.0009 |
| Eccentricity of gear-bearing center-distance | 0.0009 | -0.0009 |

| Center-distance for bores (machining tolerance) | Max. 5.0010 | Min. 5.0000 |
| Eccentricity of pinion bearing | 0.0009 | -0.0009 |
| Eccentricity of gear-bearing center-distance | 0.0009 | -0.0009 |

| Max. 5.0028 | Min. 4.9982 |

| B. Calculation of theoretical maximum and minimum center-distance (assuming reaction forces acting along line of centers; values from above) | Max. 5.0028 | Min. 4.9982 |

| Center-distance of bores (machining tolerance) | Max. 5.0010 | Min. 5.0000 |

(Continued)
TABLE 4.40 (Continued)
Summary of Calculations for Case I

<table>
<thead>
<tr>
<th>Description</th>
<th>Calculation</th>
<th>Maximum</th>
<th>Minimum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eccentricity, radial component of pinion bearing</td>
<td>(0.0009) ( \sin 20^\circ ) - (0.00015) ( \sin 20^\circ )</td>
<td>0.0003</td>
<td>–</td>
</tr>
<tr>
<td>Eccentricity, radial component of gear bearing</td>
<td>(0.0009) ( \sin 20^\circ ) - (0.00015) ( \sin 20^\circ )</td>
<td>0.0003</td>
<td>0.0005</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Max. 5.0016</td>
<td>Min. 5.0001</td>
</tr>
</tbody>
</table>

FIGURE 4.22  Cross Section of the First Stage of Gearing in an Aircraft-Type Actuator. The Members Refer to the Surfaces Discussed in Table 4.41.

and it is used here to establish the maximum and minimum probable center-distance. This method gives over 90% assurance that the values shown will not be exceeded.

4.4.2 MACHINE ELEMENTS THAT REQUIRE CONSIDERATION IN CRITICAL CENTER-DISTANCE APPLICATIONS

As illustrated in cases I and II, the operating center-distance for a pair of gears is made up of several elements, each of which contributes to overall center-distance. The accumulation of tolerances on each of these elements must be considered in application in which a minimum of backlash is to be established. The following is a consideration of the elements that have the largest contribution to variations in center distance:

- **Rolling-element bearings.** Ball-and-roller bearings consist of three major elements that contribute to backlash and to changes in center-distance. Some of these are illustrated in case II. The outer race has machining tolerances that cause eccentricity of the axes of the inner raceway and the outer
TABLE 4.41
Summary of Calculations for Case II

<table>
<thead>
<tr>
<th>No.</th>
<th>Item Discussed in Footnotes</th>
<th>Surface Designation (see Figure 4.22)</th>
<th>Tolerance (Clearance), in. (1)</th>
<th>Equivalent Change in Center Distance (3)</th>
<th>Min. Center-Distance Col. ((3) \times 10^{-6}) (4a)</th>
<th>Max. Center-Distance Col. ((3) \times 10^{-6}) (4b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Fit of motor shaft in inner race of ball bearing(^a)</td>
<td>(1)</td>
<td>0.0000</td>
<td>0.0000</td>
<td>-0.00</td>
<td>-0.00</td>
</tr>
<tr>
<td>2.</td>
<td>Eccentricity of inner ring bearing(^b)</td>
<td>(1−2)</td>
<td>0.0002</td>
<td>0.0001</td>
<td>-0.01</td>
<td>-0.01</td>
</tr>
<tr>
<td>3.</td>
<td>Radial clearance in bearing(^c)</td>
<td>(2−3)</td>
<td>0.0004−0.0000</td>
<td>0.0002</td>
<td>-0.04</td>
<td>+0.04</td>
</tr>
<tr>
<td>4.</td>
<td>Eccentricity of outer ring of bearing(^d)</td>
<td>(3−4)</td>
<td>0.0004</td>
<td>0.0002</td>
<td>-0.04</td>
<td>-0.04</td>
</tr>
<tr>
<td>5.</td>
<td>Fit of outer race of bearing into motor end shield(^e)</td>
<td>(4)</td>
<td>0.0008−0.0000</td>
<td>0.0004−0.0000</td>
<td>-0.16</td>
<td>-0.16</td>
</tr>
<tr>
<td>6.</td>
<td>Concentricity of axis of bore with axis of rabbet on end shield(^f)</td>
<td>(4−5)</td>
<td>0.0020</td>
<td>0.0010</td>
<td>-1.00</td>
<td>-1.00</td>
</tr>
<tr>
<td>7.</td>
<td>Clearance between rabbet of motor end shield and bore of actuator end plate(^g)</td>
<td>(5)</td>
<td>0.0025</td>
<td>0.00125</td>
<td>-1.56</td>
<td>-1.56</td>
</tr>
<tr>
<td>8.</td>
<td>Concentricity of motor bore in actuator end plate and rabbet(^h)</td>
<td>(5−6)</td>
<td>0.0010</td>
<td>0.0005</td>
<td>-0.25</td>
<td>-0.25</td>
</tr>
<tr>
<td>9.</td>
<td>Distance between axis of gear shaft bore and rabbet in actuator housing(^i)</td>
<td>(6−7)</td>
<td>0.0020</td>
<td>0.0010</td>
<td>-1.00</td>
<td>-1.00</td>
</tr>
<tr>
<td>10.</td>
<td>Clearance between bore for bearing and bearing outer race(^e)</td>
<td>(7)</td>
<td>0.0008</td>
<td>0.0004</td>
<td>-0.16</td>
<td>-0.16</td>
</tr>
</tbody>
</table>

(Continued)
### TABLE 4.41 (Continued)
#### Summary of Calculations for Case II

<table>
<thead>
<tr>
<th>No.</th>
<th>Item Discussed in Footnotes</th>
<th>Surface Designation (see Figure 4.22)</th>
<th>Tolerance (Clearance), in.</th>
<th>Equivalent Change in Center Distance</th>
<th>Min. Center-Distance Col. ((3)^c \times 10^6 )</th>
<th>Max. Center-Distance Col. ((3)^c \times 10^6 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>11.</td>
<td>Eccentricity, outer race of bearing (d )</td>
<td>(7–8)</td>
<td>0.0004</td>
<td>0.0002</td>
<td>−0.04</td>
<td>−0.04</td>
</tr>
<tr>
<td>12.</td>
<td>Radial clearance in bearing (c )</td>
<td>(8–9)</td>
<td>0.0004–0.0000</td>
<td>0.0002–0.0000</td>
<td>−0.04</td>
<td>−0.04</td>
</tr>
<tr>
<td>13.</td>
<td>Eccentricity, inner race of bearing (b )</td>
<td>(9–10)</td>
<td>0.0002</td>
<td>0.0001</td>
<td>−0.01</td>
<td>−0.01</td>
</tr>
<tr>
<td>14.</td>
<td>Fit of gear shaft in inner race of bearing (a )</td>
<td>(10)</td>
<td>0.0000</td>
<td>0.0000</td>
<td>−0.00</td>
<td>−0.00</td>
</tr>
</tbody>
</table>

**Note:** The maximum values of center-distance (Columns 4b and 5b) are based on the assumption that the separating forces between pinion and gear act to hold the shafts at their maximum separation within the limits of bearing clearance.

The minimum values of center-distance (Columns 4a and 5a) are based on the assumption that the external forces on gear and pinion act to hold the shafts at their minimum separation within the limits of bearing clearance.

The minimum center-distance (Column 4) is based on the assumption that all parts, made to their maximum clearances, are assembled to achieve the minimum possible center-distance. The maximum center-distance (Column 5) is based on the assumption that all the parts, made to their maximum clearances, are assembled to achieve the maximum possible center distance.

Column (1) shows the surface designated on Figure 4.22 by a number in a circle; (2) shows the tolerance (usually on a diameter); (3) shows the amount this tolerance can contribute to center-distance (±); (4) shows the square of the tolerance contributing to the minimum center-distance; and (5) shows the square of the tolerance contributing to the maximum center-distance. The values in columns (4) and (5) are added algebraically and their square roots are shown at the foot of the column.

**a** Most bearing catalogues indicate bearing and shaft tolerances with will produce an interference fit. This fit cannot, therefore, contribute to the maximum center-distance at which the gears will operate.

**b** Most bearing catalogues indicate a tolerance on runout between the bore and the raceway of the inner ring. This eccentricity contributes to the instantaneous center-distance when the inner face is the moving portion of the bearing.

**c** Most bearing catalogues include a certain radial clearance between the inner and the outer race of a bearing. This clearance usually allows an increase in center-distance due to the reaction between the two meshing gears.

**d** Most bearing catalogues indicate a tolerance on runout between the raceway and the outer diameter of the outer ring. This eccentricity contributes to the center-distance at which the gears will operate. This value will change during running since the outer race is often permits to creep.

**e** Most bearing catalogues indicate a bore tolerance on the hole that the bearing is fitted in. Usually the tolerance will provide clearance between the bearing and housing. When clearance exists, the gear reaction acts to increase the center-distance to the limit allowed by the clearance.

**f** Many manufacturers of pilot-mounted motors specify a value of runout between the motor shaft and the pilot surface. It is customary to measure this by holding the motor shaft fixed (as between centers) and to rotate the motor about its shaft. A dial indicator running on this surface will show runout (twice eccentricity).

**g** A clearance fit is usually specified on the hole into which the pilot of the motor is to fit. This clearance allows the motor to be shifted somewhat and, as a result, has a direct effect on center-distance. The value shown assumes a tolerance of \(+0.0005\) in. on the diameter of the pilot on the motor.

**h** In an assembly, such as shown in this example, a tolerance must be placed on the location of the bore to receive the motor relative to the rabbet on the end plate.

**i** This, in effect, is the same type of tolerance as discussed under “g” above.
bore. Depending on how this member is installed, the center-distance established by the bores in which this element is fitted will be increased or decreased. It is customary to let this element creep; thus, the eccentricity will go through all positions. There is also a tolerance on the outer diameter of this member. This tolerance, plus the tolerance on the bore in which this element is fitted, can cause varying degrees of looseness and therefore changes in operating center-distance.

The inner race has machining tolerances that cause eccentricity in the same manner, as in the outer race member. In cases where the shaft rotates, it is customary to use a light interference fit between the inner race and the shaft. In such cases, the center of the gear will move in a path about the center of the ball or roller path of the bearing. The eccentricity of the gear will be that of the inner race of the bearing. This will cause a once-per-revolution change in center-distance.

The clearance in the bearing is controlled by the selection of the diameter of the balls. This clearance allows greater or lesser changes in center-distance of the gears supported by the bearings

- **Journal bearings.** Journal bearings can change the center-distance at which the gear was intended to operate by an amount that is a function of the clearance designed into the bearing, its direction of rotation, the speed of rotation, and the lubricant used. The change from the distance actually established by the distance between the centers of the bores is due to the oil wedge developed. Handbooks give convenient methods of evaluating the eccentricity that can be expected of a given bearing design. When necessary, these refinements can be introduced into the calculations illustrated by cases I and II

- **Sleeve bearings.** The term sleeve bearings is used here to identify the journal-type bearing in which a sleeve of some bearing material is pushed into the gear casing or frames. The tolerance of the eccentricity of the outside diameter and the inside bearing surface is the element to consider in this type of bearing. In some cases, the clearance between the sleeve and the shaft is affected because of the interference fit between the sleeve and the bore in the casing

- **Casing bores.** The distance between bearing bores in a gear casing can be made up of several elements. In the simplest case, a single frame or casing has two holes drilled at a specified center-distance. The tolerance is selected to give the machine operator the necessary working allowance (see Figure 4.19). This tolerance has a direct effect on the operating center-distance of the gears.

In some designs, the casing is made up of several parts bolted together. One part may carry the bore for the gear shaft, and another may carry the bore for the pinion shaft. The center-distance is then made up of a series of parts each having dimensions and tolerances that can add up in various ways to give maximum and minimum center-distance. Case II illustrates this type of gearing.

### 4.4.3 Control of Backlash

In cases where it is necessary to control the amount of backlash introduced by the mountings, two courses of action may be employed. In the first, the center-distance may be made adjustable. That is, provisions may be made so that, at assembly, the centers may be moved until the desired mesh is obtained. This entails a method of moving the parts through very small distances, and then being able to fix them securely when the desired distance has been reached. Provision must also be made for assemblers to see what they are doing when adjusting the mesh. In this approach, the difference between the smallest and the largest backlash in the adjusted mesh is only the effect of total composite error (runout) in both meshing parts. Size tolerance on the teeth can be generous since this is one of the tolerances adjusted out of the mesh by assembler.

After the center-distance adjustment is completed, the parts may be drilled and doweled, although this causes difficulties if replacement gears are ever to be used on these centers.
In the second approach, very close tolerances are held both in the mountings (boring center-distance, bearings, etc.) and on the tooth thickness of the gear. This approach is used whenever interchangeability is specified and in mass production in which assembly costs is to be minimized.

4.4.4 Effects of Temperature on Center-Distance

Many gear designs consist of gears made of one material operating in mountings made of another material having a markedly different coefficient of expansion. Since it is customary to manufacture and assemble gears at room temperature of about 68°F, an analytical check of the effective center-distance present in the gears at their extremes of operating temperature is desirable.

Power gears may heat up because of the frictional losses in the mesh and may achieve an operating temperature above that of their casing. If made of the same materials as their mountings, such gears would have a larger apparent pitch diameter19 relative to the center-distance and would therefore have a smaller effective center-distance at their running temperature. Some gears, particularly those in military applications, may be subjected to extreme cold for periods of time. Often, these are steel gears operating in aluminum or magnesium casings. In this case, the center-distance will shrink to a greater degree than the apparent pitch diameter of the gears, and the resulting backlash will be less at room temperature.

If evaluating the effective center-distance of a mesh at an operating condition other than room temperature, the temperature of the gear blanks, as well as that of the mountings, which may be considerably different, must be considered, as well as the coefficient of expansion of the gear and mounting materials. There are six operating conditions in which temperature can have an important effect on gear performance:

1. Gears operating at temperatures higher than their mountings:
   a. Operating temperatures lower than assembly-room temperature
   b. Operating temperatures higher than assembly-room temperature
2. Gears operating at temperatures lower than their mountings:
   a. Operating temperatures lower than assembly-room temperature
   b. Operating temperatures higher than assembly-room temperature
3. Gears operating at temperatures essentially the same as their mountings:
   a. Operating temperatures lower than assembly-room temperature
   b. Operating temperatures higher than assembly-room temperature

(4.103) and (4.104) evaluate each of these possibilities and establish the most critical extremes at which minimum or maximum effective center-distance will occur.

The significance of these possibilities can be appreciated by considering the following service conditions. The first case is typical of power gearing under steady-state conditions. Condition (1.a) is a possibility at start-up under a cold operating environment. The case is usually a transitory condition where the gears may have to perform adjacent to an external source of heat. The third condition is most typical of control gears that do not transmit enough energy to be at a temperature greatly different from their mountings.

The procedure recommended here is to calculate a minimum and a maximum effective center-distance. These are based on the extremes of operating temperatures. From these values, the designer can calculate either a value of tooth thickness that will give the necessary operating backlash under the tightest mesh conditions, or the amounts of backlash that will be found in a given gear-set.

(4.103) and (4.104) give minimum and maximum effective center-distances:

19 Apparent pitch diameter is the diameter at which a given value of tooth thickness is found.
where:

- \( C_{\text{min}} \) is the minimum effective center-distance that occurs under temperature extreme
- \( C_d \) is the basic or nominal center-distance
- \( C't \) is the minimum tolerance on center-distance
- \( \Delta C_T \) is the change in center-distance; see note following (4.105) and (4.106)
- \( C_{\text{max}} \) is the maximum effective center-distance that occurs under temperature extreme
- \( C't' \) is the maximum tolerance on center-distance
- \( \Delta C_T \) is the change in center-distance; see note following (4.105) and Eq. (4.106)

(4.105) and (4.106) indicate the amount that the center-distance (effective) will change because the gears and their mountings shrink or expand at different rates:

\[
\Delta C'_T = \frac{D}{2} (\Delta T'_M K_M - \Delta T'_G K_G) + \frac{d}{2} (\Delta T'_M K_M - \Delta T'_G K_P)
\]  
(4.105)

\[
\Delta C''_T = \frac{D}{2} (\Delta T''_M K_M - \Delta T''_G K_G) + \frac{d}{2} (\Delta T''_M K_M - \Delta T''_G K_P)
\]  
(4.106)

Note: Compare \( \Delta C'_T \) and \( \Delta C''_T \). Assign the smallest positive number or the largest negative number to \( \Delta C_T \). Assign the largest positive number or the smallest negative number to \( \Delta C_T \). If signs are opposite, use this negative value for \( \Delta C_T \), and use the positive value for \( \Delta C_T \). These values are used in (4.103) and (4.104) to obtain effective center-distance:

\[
\Delta T'_M = T_M - T_R
\]  
(4.107)

\[
\Delta T''_M = T'_M - T_R
\]  
(4.108)

\[
\Delta T'_G = T_G - T_R
\]  
(4.109)

\[
\Delta T''_G = T'_G - T_R
\]  
(4.110)

where:

- \( K_P \) is the coefficient of expansion of pinion material, in. per in. per °F
- \( K_G \) is the coefficient of expansion of gear material
- \( K_M \) is the coefficient of expansion of mounting material
- \( T_M \) is the minimum mounting temperature (operating)
- \( T_R \) is the assembly room temperature (68°F)
- \( T'_M \) is the maximum mounting temperature (operating)
- \( T_G \) is the minimum gearing temperature (operating)
- \( T'_G \) is the maximum gearing temperature (operating)
4.4.5 MOUNTING DISTANCE

Bevel gearing, worm gearing, face gearing, and Spiroid gearing must be given close control on axial positioning (mounting distance) if good performance is to be achieved (see Figure 4.23). Most of the elements that must be given control to achieve proper center-distance must also be given close control to achieve proper mounting distance. If any of the above types of gearing can move along the axis of its shaft relative to the mating gear, the teeth will either bind or in the opposite case, have excessive backlash.

Straight-toothed bevel gears are critical in respect to axial position, since backlash and tooth bearing pattern are affected by changes in axial position of the pinion and gear. Spiral bevel gears, hypoid gears, and Spiroid gears are also critical in that they tend to “screw” into mesh in one direction of rotation, which can cause binding, and will “unscrew” in the other direction of rotation, causing backlash, unless the mounting system is stiff enough to prevent axial shift. All properly designed mountings for gears of this type have bearings capable of withstandng all axial thrust loads imposed both by the gears and by possible external loads. Occasionally a design is attempted in which the designer relies on the separating

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20 See Chapter 15 for general information on gear mounting tolerances and practices.
forces produce in the mesh to keep the gears away from a binding condition. Only a few such designs are truly successful.

Worm gearing must also be accurately positioned if of the single- or double-enveloping design. If a throated member is permitted to move axially, the frictional force developed in the mesh will tend to move the member, and the geometry of the throated design will cause a reduction in backlash, sometimes to the point of binding.

Mounting distance is usually specified for each gear member as the distance from a specific mounting surface on the gear blank, a face of a hub, for example, to the axis of the mating gear. Bevel gears are usually stamped with the correct mounting distance for the pair. This distance was established in a bevel gear test fixture at the time of manufacture.

The design of mountings for gearing requiring a mounting distance should include provisions for shimming or otherwise adjusting the position of the members at assembly. The calculation of mounting distance is accomplished in two steps. The design of the gear teeth (earlier in this chapter) includes a distance from the axes or pitch cone apex to a given surface of the gear. To this distance is added the distance to the closest bearing face that is used to provide the axial location of the gear and its shaft. Provision should be made in the design to secure the bearing seats that provide axial location at the specified mounting distances.

In the previous chapter, radial clearance in rolling-element bearing was considered. In gear designs requiring a mounting distance, the axial clearance in one bearing on each shaft must be controlled. Reference to a bearing catalogue or handbook is recommended.

**BIBLIOGRAPHY**


